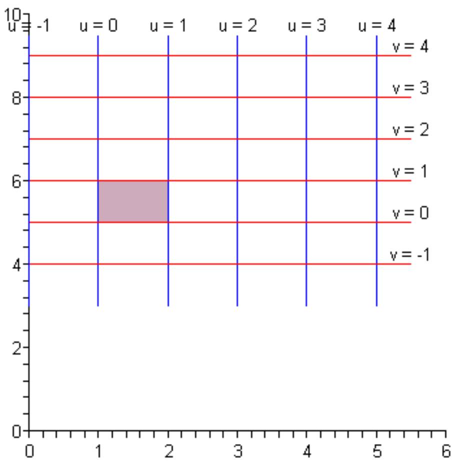
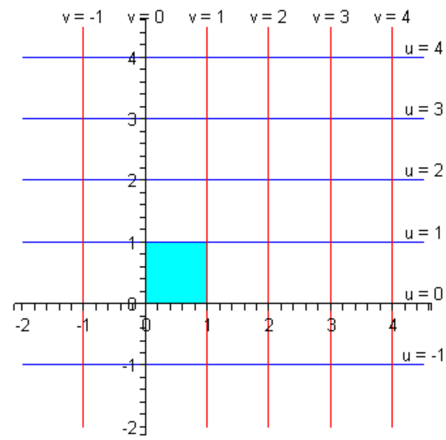
1. Section 3-1

1. $y = 4x^2$
3. $y = 2x$
5. $\frac{x^2}{16} + \frac{y^2}{9} = 1$
7. $x = \frac{y^2}{4} - 4$
9. $x^2 + y^2 = 1$

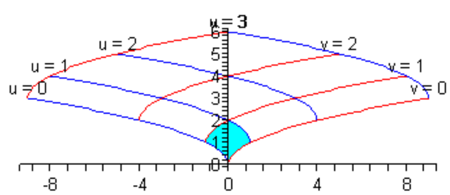
11.



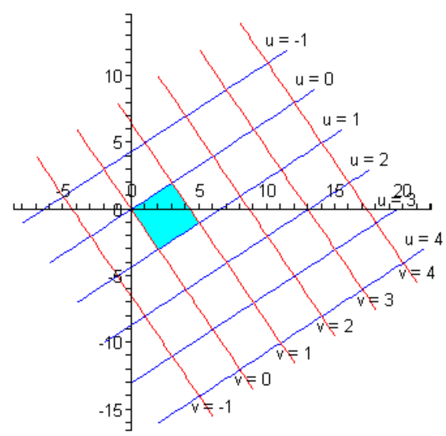
13.



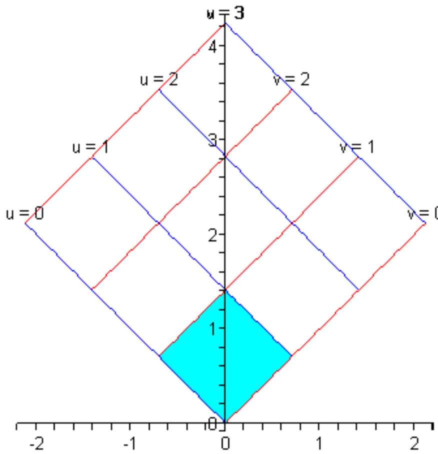
15.



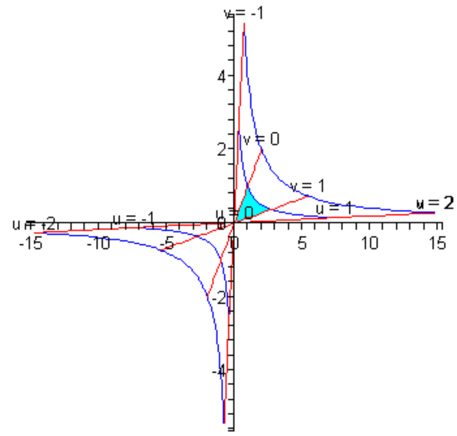
17.



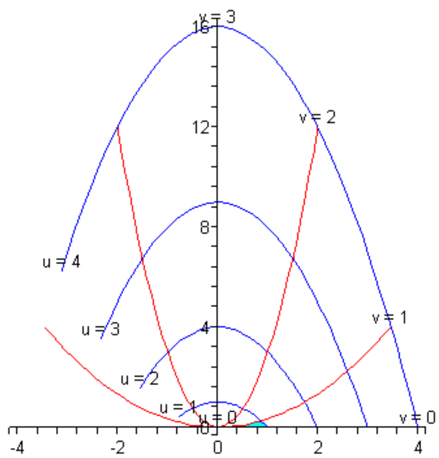
19.



21.



23.



25. $u^2 + \frac{v^2}{4} = 1$

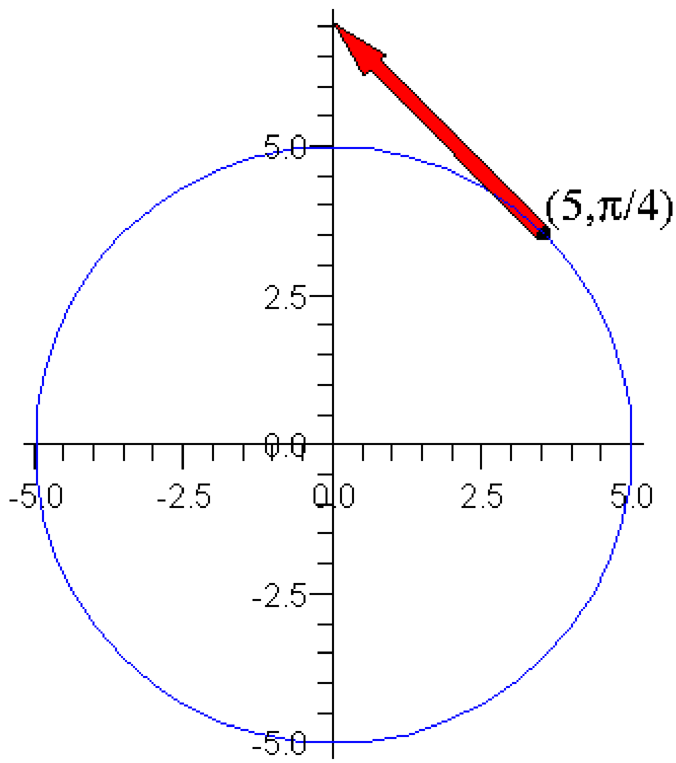
27. $u^2 - v^2 = 2$

29. $\frac{u^2}{16} + \frac{v^2}{9} = 1$

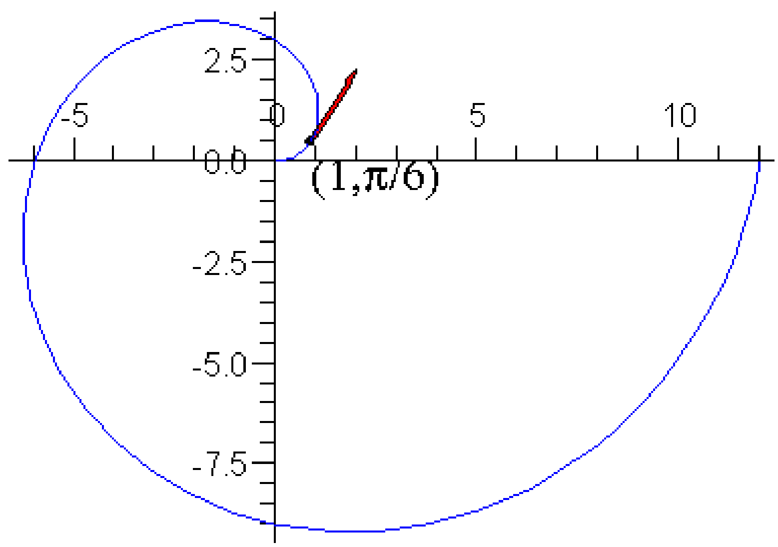
2. Section 3-2

1. a. $(-2, 0)$ b. $(-1, 0)$ c. $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ d. $\left(1, -\sqrt{3}\right)$

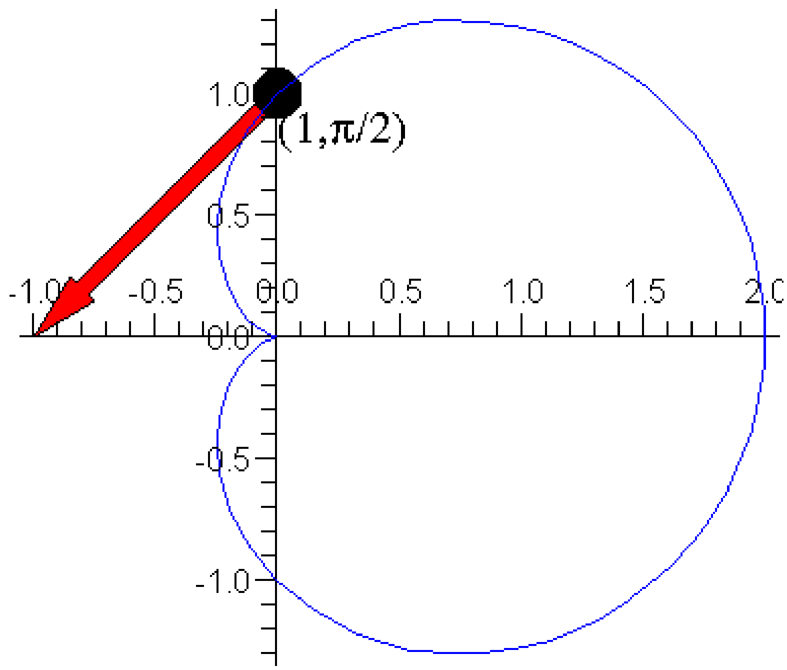
3. $\mathbf{v}\left(\frac{\pi}{4}\right) = \left\langle \frac{-5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$



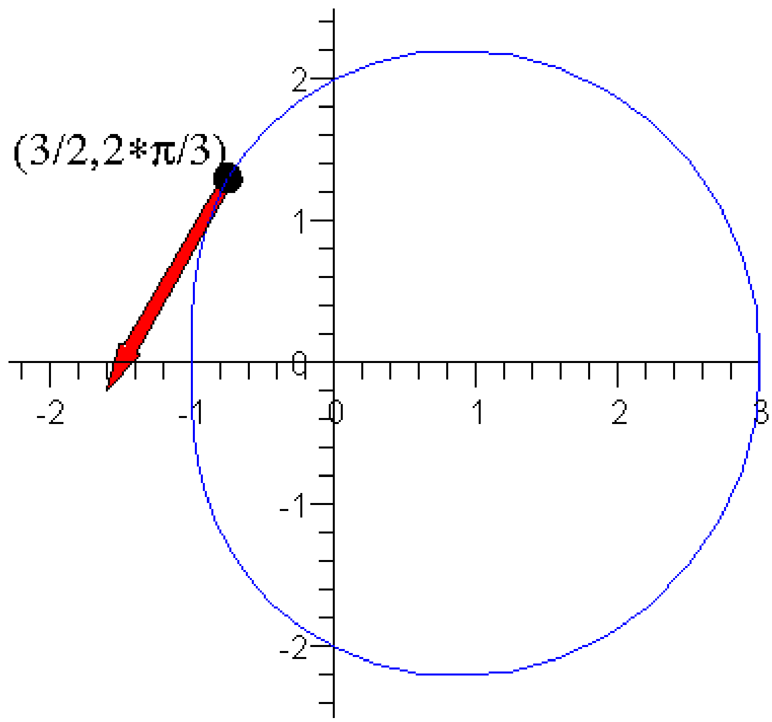
5. $\mathbf{v}\left(\frac{\pi}{6}\right) = \left\langle \frac{3\sqrt{3}}{\pi} - \frac{1}{2}, \frac{3}{\pi} - \frac{\sqrt{3}}{2} \right\rangle$



7. $\mathbf{v}\left(\frac{\pi}{2}\right) = \langle -1, -1 \rangle$



9. $\mathbf{v}\left(\frac{\pi}{6}\right) = \left\langle \frac{-\sqrt{3}}{2}, \frac{-3}{2} \right\rangle$



11. $r = 4$

13. $r = \sec(\theta)$

15. $r = \dfrac{2}{\sin(\theta) - 3\cos(\theta)}$

17. $r = \sec(\theta)\tan(\theta)$

19. $r = \dfrac{2}{1 - \cos(\theta)}$

21. $r = \sin(\theta) + \sqrt{\sin^2(\theta) + 3}$

23. $r = 2\cos(\theta)$

25. $r = \sqrt{2\sec(\theta)\csc(\theta)}$

27. $p = 3,\ \varepsilon = \frac{3}{4}$

29. $p = 2,\ \varepsilon = 0.2$

31. $p = 2,\ \varepsilon = 0.5$

33. $p = 5,\ \varepsilon = 0.6$

3. Section 3-5

1. $\mathbf{w} = \langle 1, 2 \rangle$, $J = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\mathbf{z} = \langle 3, -1 \rangle$
3. $\mathbf{w} = \langle 1, 3 \rangle$, $J = \begin{bmatrix} 2uv & u^2 \\ v^2 & 2uv \end{bmatrix}$, $\mathbf{z} = \langle 36, 108 \rangle$
5. $\mathbf{w} = \langle 1, 0 \rangle$, $J = \begin{bmatrix} \sec(v) & u\sec(v)\tan(v) \\ \tan(v) & u\sec^2(v) \end{bmatrix}$, $\mathbf{z} = \langle 1, 0 \rangle$
7. $dA = 2dudv$
9. $dA = \left(4u^2 + 4v^2\right) dudv$
11. $dA = 2\,|u|\,dudv$
13. $dA = 6\,|u|\,dudv$
15. $dA = |\cos(2u)|\;dudv$
17. $dA = \left(\sin^2(u) + \sinh^2(v)\right) dudv$
19. $dA = 2dudv$
21. $dA = dudv$
23. $dA = \left(4u^2 + 4v^2\right) dudv$
25. $dA = \left|\sinh^2(v) - \sin^2(u)\right| dudv$

4. Section 3-4

1. $x^2 + y^2 = z^2$
3. $x^2 + y^2 + z^2 = 1$
5. $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$
7. $x^2 - y^2 + z^2 = 1$
9. $x^2 + y^2 - z^2 = 1$
11. $\left(x^2 + y^2\right) z^2 = 1$

13.	$\mathbf{r}_u = \langle -\sin(u)\sin(v), 0, \cos(u)\sin(v) \rangle$	$\mathbf{r}_v = \langle \cos(u)\cos(v), -\sin(v), \sin(u)\cos(v) \rangle$	orthogonal
15.	$\mathbf{r}_u = \langle 0, -\sin(u)\cos(v), \cos(u)\cos(v) \rangle$	$\mathbf{r}_v = \langle \cos(v), -\cos(u)\sin(v), -\sin(u)\sin(v) \rangle$	orthogonal
17.	$\mathbf{r}_u = \left\langle 1, \dfrac{-\sin(v)\,u}{\sqrt{1-u^2}}, \dfrac{-\cos(v)\,u}{\sqrt{1-u^2}} \right\rangle$	$\mathbf{r}_v = \sqrt{1-u^2}\quad \langle 0, \cos(v), -\sin(v) \rangle$	orthogonal
19.	$\mathbf{r}_u = \dfrac{\left\langle 2\left(v^2 - u^2 + 1\right), -4uv, 4u \right\rangle}{\left(u^2 + v^2 + 1\right)^2}$	$\mathbf{r}_v = \dfrac{\left\langle -4uv, 2\left(v^2 - u^2 + 1\right), 4v \right\rangle}{\left(u^2 + v^2 + 1\right)^2}$	orthogonal

21. $\mathbf{r}(u,v) = \langle u, u\sin(v), u\cos(v) \rangle$ orthogonal
    $\mathbf{r}_u = \langle 1, \sin v, \cos v \rangle$ $\quad \mathbf{r}_v = \langle 0, u\cos v, -u\sin v \rangle$
23. $\mathbf{r}(u,v) = \left\langle u, \left(u - u^2\right)\sin(v), \left(u - u^2\right)\cos(v) \right\rangle$ orthogonal
    $\mathbf{r}_u = \langle 1, (1-2u)\sin v, (1-2u)\cos v \rangle$ $\quad \mathbf{r}_v = \left\langle 0, \left(u - u^2\right)\cos v, -\left(u - u^2\right)\sin v \right\rangle$
25. $\mathbf{r}(u,v) = \langle u, \cosh(u)\sin(v), \cosh(u)\cos(v) \rangle$ orthogonal
    $\mathbf{r}_u = \langle 1, \sinh u \sin v, \sinh u \cos v \rangle$ $\quad \mathbf{r}_v = \langle 0, \cosh u \cos v, -\cosh u \sin v \rangle$

5. Section 3-5

1. a. $\left(\frac{3}{2}, \frac{3}{2}\sqrt{3}, 3\right)$ b. $(0,7,0)$ c. $(5,0,0)$ d. $(-4,0,-2)$

3. a. $\left(-\frac{3}{2}\sqrt{3}, 0, \frac{3}{2}\right)$ b. $\left(\frac{7}{2}\sqrt{2}, \frac{7}{2}\sqrt{2}, 0\right)$ c. $(-1,0,0)$ d. $(0,0,5)$

In 5-11, substitute the value for r into the parameterization

$$\mathbf{r}(\theta, z) = \langle r\cos(\theta), r\sin(\theta), z\rangle$$

In 13-19 amd 23, substitute the value for ρ into the parameterization

$$\mathbf{r}(\phi, \theta) = \langle \rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi)\rangle$$

In 25-29, substitute the value for r into the parameterization

$$\mathbf{r}(t) = \langle r\cos(t), r\sin(t), r\rangle$$

5. $r = 5,$
7. $r = \sqrt{z^2+1}$
9. $r = \dfrac{2}{3\cos(\theta)+4\sin(\theta)}$
11. $r = z$
13. $\rho = 5$
15. $\rho = \csc(\phi)\sec(\theta)$
17. $\rho = \sqrt{-\sec(2\phi)}$
19. $\rho = \dfrac{1}{\cos(\phi)+2\sin(\phi)\sin(\theta)}$
21. $\phi = \pi/4,\ \mathbf{r}(\rho,\theta) = \frac{\rho}{\sqrt{2}}\langle\cos(\theta), \sin(\theta), 1\rangle$
23. $\rho = \sec(\phi)\sin(2\theta)$
25. $r = \dfrac{1}{1-\frac{1}{2}\cos(\theta)}$
27. $r = \dfrac{2}{1-\cos(\theta)}$
29. $r = \dfrac{1}{1-2\cos(\theta)}$

6. Section 3-6

1. $z = \frac{11}{3} - \frac{1}{3}x - \frac{1}{3}y$
3. $z = -\frac{1}{2}x + 3 - \frac{1}{4}y$
5. $3x + 4y + 2z = 13$
7. $z = x + y - 1$
9. $z = -x + 2 - y$
11. $z = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$
13. $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$
15. $z = \left(\frac{\sqrt{3}}{6} + \frac{1}{2}\right)x + \left(\frac{1}{6} - \frac{\sqrt{3}}{2}\right)y$
17. $z = \sqrt{2} - \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$
19. $z = e^{-2}y + 2e^{-1}$
21. $z = 4x - y - 4$
23. $z = 3x + 2y + 1$
25. $z = x + 4y$

$$\langle \sin(t), \cos(t), 0\rangle$$
$$\langle \cos(t), -\sin(t), \rangle$$
$$\langle 0, 0, \sin(t)\cos(0)\rangle$$

7. Section 3-7

1. $ds^2 = 2du^2 + dv^2$
3. $ds^2 = v^2 du^2 + 2dv^2$
5. $ds^2 = du^2 + \sin^2(u)\, dv^2$
7. $ds^2 = \cosh^2(v)\, du^2 + \left(2\cosh^2 v - 1\right) dv^2$
9. 2π
11. 1
13. 2π
15. $1.931\,9$
17. 2π
19. π
21. $\boldsymbol{\rho}'' = 0$, $\boldsymbol{\rho}(t)$ is a straight line, so is a geodesic
23. $\boldsymbol{\rho}'' \cdot \mathbf{r}_u = 32t + 24$, $\quad \boldsymbol{\rho}'' \cdot \mathbf{r}_v = -(4t+3)$, not a geodesic
25. $\boldsymbol{\rho}'' \cdot \mathbf{r}_u = 0$, $\quad \boldsymbol{\rho}'' \cdot \mathbf{r}_v = 0$, is a geodesic
27. $\boldsymbol{\rho}'' \cdot \mathbf{r}_u = 0$, $\quad \boldsymbol{\rho}'' \cdot \mathbf{r}_v = \sinh(t)\cosh(t)$, is not a geodesic
29. $\mathbf{r}(t) = \left[\sin(t), \sin(t), \cos(t)\sqrt{2}\right]$, distance $= \frac{\pi\sqrt{2}}{2}$
31. $\mathbf{r}(t) = \langle 2,2,1\rangle \cos(t) + \left\langle \frac{2}{\sqrt{17}}, -\frac{7}{\sqrt{17}}, \frac{10}{\sqrt{17}} \right\rangle \sin(t)$
    distance $= \cos^{-1}\left(\frac{8}{9}\right) \approx 0.47588$

8. Section 3-8

1. $\kappa_n(\theta) = \cos^2(\theta)$, $\kappa_1 = 0$, $\kappa_2 = 1$, *Gaussian flat*
3. $\kappa_n(\theta) = 0$, *flat*, minimal, (it's a plane!)
5. $\kappa_n(\theta) = \dfrac{2\sin^2(\theta)}{(1+4v^2)^{3/2}}$, $\kappa_1 = 0$, $\kappa_2 = \dfrac{2}{(1+4v^2)^{3/2}}$, Gaussian flat
7. $\kappa_n(\theta) = \dfrac{\cos^2(\theta)}{\cosh^2(u)}$, $\kappa_1 = \dfrac{1}{\cosh^2(u)}$, $\kappa_2 = 0$, Gaussian flat
9. $\kappa_n(\theta) = \dfrac{-\sin(2\theta)}{1+u^2}$, $\quad \kappa_1 = \dfrac{-1}{1+u^2}$, $\quad \kappa_2 = \dfrac{1}{1+u^2}$, Minimal
11. $\kappa_n(\theta) = \dfrac{\sin^2(\theta)}{u\sqrt{2}}$, $\quad \kappa_1 = \frac{1}{u\sqrt{2}}$, $\quad \kappa_2 = 0$, Gaussian flat
13. $\kappa_n(\theta) = \left(2\cos^2(\theta) - 1\right)\operatorname{sech}^2(v)$, $\quad \kappa_1 = \operatorname{sech}^2(v)$, $\quad \kappa_2 = -\operatorname{sech}^2(v)$, Minimal
15. $\kappa_n(\theta) = -\sin(2\theta)\operatorname{sech}^2(v)$, $\quad \kappa_1 = \operatorname{sech}^2(v)$, $\quad \kappa_2 = -\operatorname{sech}^2(v)$, Minimal
17. $K = -4$
19. $K = 0$
21. $K = \dfrac{-2}{v^2\left(4v^2+1\right)^2}$
23. $K = \dfrac{\cos u}{2 + \cos u}$