# Practice Test

Chapter 3 Name ______________________________

**Instructions.** Show your work and/or explain your answers. (**Note:** Concepts from "DIFF GEOM" sections included only in the last problem).

1. Find the equation of the tangent plane to the level surface
$$x + z^2 = y + 1$$
at the point $(2, 2, 1)$.

2. Find the level surface representation of the parametric surface
$$\mathbf{r}(u, v) = \left\langle v\sin(u), v^2, v\cos(u) \right\rangle$$

3. Find the level surface representation of the parametric surface
$$\mathbf{r}(u, v) = \left\langle e^u \cosh(v), e^u \sinh(v), e^{-u} \right\rangle$$

4. Find the parametric equation of the tangent plane to the parametric surface
$$\mathbf{r}(u, v) = \left\langle v\sin(u), v^2, v\cos(u) \right\rangle$$
at $(u, v) = \left(\frac{\pi}{4}, 1\right)$.

5. Find the image of the unit square under the coordinate transformation
$$T(u, v) = \left\langle u^2 - v^2, 2uv \right\rangle$$

6. Find the matrix of rotation through an angle of $\theta = 45^\circ$. Then use this to rotate the line $v = u + 1$ through an angle of $45^\circ$.

7. Convert the following into polar coordinates and solve for $r$ :
$$y = 3x + 1$$

8. Convert the following into polar coordinates and solve for $r$ :
$$x^2 + y^2 = x + y$$

9. Sketch the graph of $r = 4\pi - \theta$ in polar coordinates when $\theta$ is in $[0, 4\pi]$. Then find and sketch the tangent vector to the curve when $\theta = \pi$

10. Find the Jacobian determinant and area differential of the coordinate transformation
$$T(u, v) = \left\langle u - v, u^2 + v^2 \right\rangle$$

11. What is the unit surface normal for the surface $x^2 + y^2 + z^2 = 2x$? What is the unit surface normal for the surface in *cylindrical* coordinates?

12. Find the pullback of the surface $x^2 + y^2 + z^2 = 2x$ into spherical coordinates, and then use the result to construct a parameterization of the surface.

13. Use the fundamental form of the plane in polar coordinates to find the length of the polar curve $r = e^{-\theta/4}$ , $\theta$ in $[0, 2\pi]$.

14. Find the fundamental form of the surface $\mathbf{r}(u,v) = \langle v\sin(u), v\cos(u), v\rangle$. Then use it to compute the arclength of

$$v = \sin\left(\frac{u}{\sqrt{2}}\right), \quad u \ in \ \left[0, \frac{\pi}{4}\right]$$

15. DIFF GEOM: For the right circular cone, $\mathbf{r}(r,\theta) = \langle r\cos(\theta), r\sin(\theta), r\rangle$, do the following:

    (a) Show that curves of the form $\theta = k$ for $k$ constant are geodesics on the cone. What are these curves?

    (b) Find the fundamental form of the cone and calculate the shortest distance between the points with coordinates $(r,\theta) = (1,\pi)$ and $(r,\theta) = (3,\pi)$.

    (c) What are the principal curvatures of the surface?

    (d) What are the mean and Gaussian curvatures of the surface? Do you obtain the same Gaussian curvature if you use the theorem Egregium?