Chapter 2: Answers to Selected Odd Exercises

Section 2-1:

1. $dom\,(f) = \{(x,y)\ \mid y \geq 2x - 1\}\,,\quad closed, connected, unbounded$
3. $dom\,(f) = \{(x,y)\ \mid x \geq 0\ and\ y \geq 0\}$ *closed*, connected, unbounded
5. $dom\,(f) = \{(x,y)\ \mid x \neq 0, y \neq 0,\ x^2 + y^2 < 1\}$ *open*, connected, bounded
7. $dom\,(f) = \{(x,y)\ \mid x \neq 0, y > 0,\ x^2 + y^2 < 4,\ x^2 + y^2 \neq 3\}$ open, not connected, bounded
9. $dom\,(f) = \{(x,y)\ \mid y > 0\}$ *open*, connected, unbounded
11. $dom\,(f) = \{(x,y)\ \mid x > 0\ and\ y < 1\}$ *open*, connected, unbounded
13. $dom\,(f) = \{(x,y)\ \mid y \neq x,\ y \neq -x\}$open, not connected, unbounded
15. $dom\,(f) = \{(x,y)\ \mid -\pi/2 \leq x - y \leq \pi/2\}$ *closed*, connected, unbounded

Section 2-2:

1. 6
3. -2
5. 8
7. 0
9. 1

11. try $y = 0$ and $x = 0$
13. try $y = 0$ and $x = 0$
15. try $y = 0$ and $y = x$
17. try $y = 0$ and $x = y^2$
19. try $x = 0$ and $y = 0$
21. try $x = \pi$ and $y = 0$

23. *cont* for $y \neq x$
25. cont for $x > 0, y \neq n\pi/2,\ n\ odd$

Section 2-3:

1. $f_x(x,y) = 2x, \quad f_y(x,y) = 3y^2$
3. $f_x(x,y) = 2x + 4y, \quad f_y(x,y) = 4x + 8y$
5. $f_x(x,y) = \sin(y), \quad f_y(x,y) = x\cos(y)$
7. $f_x(x,y) = -2x\exp(-x^2 - y^2), \quad f_y(x,y) = -2y\exp(-x^2 - y^2)$
9. $f_x(x,y) = \cos(xy) - xy\sin(xy), \quad f_y(x,y) = -x^2\sin(xy)$
11. $f_x(x,y) = y^x\ln(y), \quad f_y(x,y) = x\ y^{x-1}$
13. $f_x(x,y) = \frac{y^2-x^2}{(x^2+y^2)^2}, \quad f_y(x,y) = \frac{-2xy}{(x^2+y^2)^2}$

15. $f_{xx} = 2, \ f_{yy} = 6y, \ f_{xy} = 0$
17. $f_{xx} = 2, \ f_{yy} = 8, \ f_{xy} = 4$
19. $f_{xx} = 0, f_{yy} = -x\sin(y), f_{xy} = \cos(y)$
21. $f_{xx} = -y^3\cos xy, \ f_{yy} = -2(\sin xy)x - y(\cos xy)x^2, \ f_{xy} = -2(\sin xy)y - y^2(\cos xy)x$
23. $f_{xx} = y^x(\ln y)^2, \quad f_{yy} = x(x-1)y^{x-2}, \quad f_{xy} = xy^{x-1}\ln(y) + y^{x-1}$
25. $f_{xxy} = 0$
27. $f_{xyx} = 0$
29. $f_{xxyy} = 0$
31. $f_{xxxy} = 4y^3\sin(xy) + xy^4\cos(xy)$

Section 2-4:

No solutions necessary for 1-9

11. $u(x,t) = Pe^{k(x+t)}$
13. $u(x,y) = Pe^{k(x-y)}$
15. $u(x,y) = P\exp(x^2\omega^2)\exp(-\omega^2 y)$
17. $u(x,t) = Pe^{-\omega^2 x}e^{(1+\omega^2)t}$
19. $u(x,y) = (A_1\cos(\omega x) + B_1\sin(\omega x))(A_2\cosh(\omega x) + B_2\sinh(\omega x))$
21. $u(x,y) = Pe^{-\omega^2 t}\left[A\cos\left(x\sqrt{\omega^2+1}\right) + B\sin\left(x\sqrt{\omega^2+1}\right)\right]$

23. $u(x,t) = \left(\frac{k}{\omega}\frac{Pe^{\omega kt}+1}{Pe^{\omega kt}-1}\right)(-\omega^2 x + C)$

Section 2-5:

1. $L(x,y) = 2x + 12y - 17$
3. $L(x,y) = 3x$
5. $L(x,y) = x + y - 2 + \ln 2$
7. $L(x,y) = -\pi^2 y + \pi^3$
9. $L(x,y) = 3x + 7y + 1$
11. $L(x,y) = \pi x + y - 2\pi$
13. $L(x,y) = 3y - 4$
15. $Q(x,y) = -15 + 12y + 2(x-1) + (x-1)^2 + 6(y-2)^2$
17. $Q(x,y) = x^2 + xy + 3x$
19. $Q(x,y) = 3x - 2y + 1$
21. $Q(x,y) = \ln(2) + (x-1) + (y-1) - (y-1)(x-1)$
23. $Q(x,y) = x^2(y-\pi)$
25. $\Delta z = 0.23,\ dz = 0.22$
27. $\Delta z = 0.04196,\ dz = 0.0414$

Hessians:

15. $H(x,y) = \begin{bmatrix} 2 & 0 \\ 0 & 6y \end{bmatrix}$

17. $H(x,y) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

19. $H(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

21. $H(x,y) = \begin{bmatrix} \frac{2y^2-2x^2}{(x^2+y^2)^2} & \frac{-4xy}{(x^2+y^2)^2} \\ \frac{-4xy}{(x^2+y^2)^2} & \frac{2x^2-2y^2}{(x^2+y^2)^2} \end{bmatrix}$

23. $H(x,y) = \begin{bmatrix} 2\sin y & 2x\cos y \\ 2x\cos y & -x^2\sin(y) \end{bmatrix}$

Section 2-6:

1. $18t^{17}$
3. 0
5. $5t^4$
7. $2\cos(2t) - \sin(2t) + 3t^2$
9. -1
11. $\frac{\partial w}{\partial u} = 4u^3v^2 + 6(u+v)^5$, $\frac{\partial w}{\partial v} = 2u^4v + 6(u+v)^5$
13. $\frac{\partial w}{\partial u} = 0$, $\frac{\partial w}{\partial v} = 0$
15. $w_x = \frac{-x}{t^2}e^{-x^2/2t} - \frac{2x}{t^3}e^{-x^2/t},\ w_t = -\frac{1}{t^2}e^{-x^2/(2t)} + \frac{1}{2t^3}x^2e^{-x^2/(2t)} - \frac{2}{t^3}e^{-x^2/t} + \frac{1}{t^4}x^2e^{-x^2/t}$
17. $\sin(t^3) + \int_0^t u^2\cos(u^2t)\,du$
19. 0

21. $\nabla f = \langle 2x, 3y^2 \rangle$
$df/dt = 4t^3 + 9t^8$
23. $\nabla f = \langle 2x + 2y, 2x \rangle$
$df/dt = -\sin 2t + 2\cos 2t$
25. $\nabla f = \langle \cos(y), -x \sin(y) \rangle$
$df/dt = \cos(2t)$
27. $\nabla f = \langle -\exp(-x^2), \exp(-y^2) \rangle$
$= e^{-\sin^2 t} \cos t + e^{-\cos^2 t} \sin t$

Section 2-7:

19. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, \quad (D_{\mathbf{u}} f)(1,1) = \sqrt{2}$
21. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, \quad (D_{\mathbf{u}} f) = 3\sqrt{2}$
23. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, \quad (D_{\mathbf{u}} f) = \sqrt{2}$

Section 2-8:

1. min at $(0, 0,)$
3. *saddle* at $(-2, 1)$
5. *saddle* at $(2, 1)$
7. min at $\left(\frac{13}{12}, \frac{-3}{4}\right)$
9. min at $(2, 0)$, saddle at $(0, 0)$
11. *saddle* at $(0, 0)$, max at $(-1, -1)$
13. *saddle* at $(0, 0)$, min at $(1, 1)$ and $(-1, -1)$
15. *saddles* at $(0, n\pi)$ for any integer n
17. *saddle* at $(0, 0)$
19. min at $(-1, 0)$, max at $(1, 0)$

21. $y = x$
23. $y = 6.15 - 0.5x$
25. $y = 74 + 2.4x$

Section 2-9:

1. min at $\left(-\frac{3}{13}\sqrt{13}, -\frac{2}{13}\sqrt{13}\right)$
   max at $\left(\frac{3}{13}\sqrt{13}, \frac{2}{13}\sqrt{13}\right)$
3. min at $\left(-\sqrt{5}, 2\sqrt{5}\right)$
   max at $\left(\sqrt{5}, -2\sqrt{5}\right)$
5. min at $(1, 0)$, $(-1, 0)$
   max at $(0, 1)$, $(0, -1)$
7. min at $\left(\frac{\pm 1}{\sqrt{2}}, \frac{\pm 1}{\sqrt{2}}\right), \left(\frac{\pm 1}{\sqrt{2}}, \frac{\mp 1}{\sqrt{2}}\right)$
   max at $(1, 0)$
9. min at $(0, 1), (0, -1)$
   max at $(1, 0), (-1, 0)$

Find the point(s) on the given curve closest to the origin.

11. $\left(\frac{1}{2}, \frac{1}{2}\right)$
13. $(1, 1)$, $(-1, -1)$
15. $(-2, 1), (2, 1)$