

Chapter 2: Answers to Selected Odd Exercises

Section 2-1:

1. $\text{dom}(f) = \{(x, y) \mid y \geq 2x - 1\}$, *closed, connected, unbounded*
3. $\text{dom}(f) = \{(x, y) \mid x \geq 0 \text{ and } y \geq 0\}$ *closed, connected, unbounded*
5. $\text{dom}(f) = \{(x, y) \mid x \neq 0, y \neq 0, x^2 + y^2 < 1\}$ *open, connected, bounded*
7. $\text{dom}(f) = \{(x, y) \mid x \neq 0, y > 0, x^2 + y^2 < 4\}$ *open, not connected, bounded*
9. $\text{dom}(f) = \{(x, y) \mid y > 0\}$ *open, connected, unbounded*
11. $\text{dom}(f) = \{(x, y) \mid x > 0 \text{ and } y < 1\}$ *open, connected, unbounded*
13. $\text{dom}(f) = \{(x, y) \mid y \neq x, y \neq -x\}$ *open, not connected, unbounded*
15. $\text{dom}(f) = \{(x, y) \mid -\pi/2 \leq x - y \leq \pi/2\}$ *closed, connected, unbounded*

17. $y = 1, y = 2, y = 3, y = 4$
19. $x^2 + y^2 = 8, x^2 + y^2 = 7, x^2 + y^2 = 6, x^2 + y^2 = 5$
21. $y = 4x - 1, y = 4x - 2, y = 4x - 3, y = 4x - 4$
23. $xy = 0$ (x and y axes), $xy = -1, xy = -2, xy = -3$
25. $y = e^x, y = 2e^x, y = 3e^x, y = 4e^x$

Section 2-2:

1. 6
3. -2
5. 8
7. 0
9. 1

11. try $y = 0$ and $x = 0$
13. try $y = 0$ and $x = 0$
15. try $y = 0$ and $y = x$
17. try $y = 0$ and $x = y^2$
19. try $x = 0$ and $y = 0$
21. try $x = \pi$ and $y = 0$

23. *cont* for $y \neq x$
25. *cont* for $x > 0, y \neq n\pi/2, n$ odd

Section 2-3:

1. $f_x(x, y) = 2x, \quad f_y(x, y) = 3y^2$
3. $f_x(x, y) = 2x + 4y, \quad f_y(x, y) = 4x + 8y$
5. $f_x(x, y) = \sin(y), \quad f_y(x, y) = x \cos(y)$
7. $f_x(x, y) = -2x \exp(-x^2 - y^2), \quad f_y(x, y) = -2y \exp(-x^2 - y^2)$
9. $f_x(x, y) = \cos(xy) - xy \sin(xy), \quad f_y(x, y) = -x^2 \sin(xy)$
11. $f_x(x, y) = y^x \ln(y), \quad f_y(x, y) = x y^{x-1}$
13. $f_x(x, y) = \frac{y^2 - x^2}{(x^2 + y^2)^2}, \quad f_y(x, y) = \frac{-2xy}{(x^2 + y^2)^2}$

15. $f_{xx} = 2, \quad f_{yy} = 6y, \quad f_{xy} = 0$
17. $f_{xx} = 2, \quad f_{yy} = 8, \quad f_{xy} = 4$
19. $f_{xx} = 0, \quad f_{yy} = -x \sin(y), \quad f_{xy} = \cos(y)$
21. $f_{xx} = -y^3 \cos xy, \quad f_{yy} = -2(\sin xy)x - y(\cos xy)x^2, \quad f_{xy} = -2(\sin xy)y - y^2(\cos xy)x$
23. $f_{xx} = y^x (\ln y)^2, \quad f_{yy} = x(x-1)y^{x-2}, \quad f_{xy} = xy^{x-1} \ln(y) + y^{x-1}$
25. $f_{xxy} = 0$
27. $f_{xyx} = 0$
29. $f_{xxyy} = 0$
31. $f_{xxxy} = 4y^3 \sin(xy) + xy^4 \cos(xy)$

Section 2-4:

No solutions necessary for 1-9

11. $u(x, t) = P e^{k(x+t)}$
13. $u(x, y) = P e^{k(x-y)}$
15. $u(x, y) = P \exp(x^2 \omega^2) \exp(-\omega^2 y)$
17. $u(x, t) = P e^{-\omega^2 x} e^{(1+\omega^2)t}$
19. $u(x, y) = (A_1 \cos(\omega x) + B_1 \sin(\omega x)) (A_2 \cosh(\omega x) + B_2 \sinh(\omega x))$
21. $u(x, y) = P e^{-\omega^2 t} [A \cos(x\sqrt{\omega^2 + 1}) + B \sin(x\sqrt{\omega^2 + 1})]$

23. $u(x, t) = \left(\frac{k P e^{\omega k t} + 1}{\omega P e^{\omega k t} - 1} \right) (-\omega^2 x + C)$

Section 2-5:

1. $L(x, y) = 2x + 12y - 17$
3. $L(x, y) = 3x$
5. $L(x, y) = x + y - 2 + \ln 2$
7. $L(x, y) = -\pi^2 y + \pi^3$
9. $L(x, y) = 3x + 7y + 1$
11. $L(x, y) = \pi x + y - 2\pi$
13. $L(x, y) = 3y - 4$
15. $Q(x, y) = -15 + 12y + 2(x - 1) + (x - 1)^2 + 6(y - 2)^2$
17. $Q(x, y) = x^2 + xy + 3x$
19. $Q(x, y) = 3x - 2y + 1$
21. $Q(x, y) = \ln(2) + (x - 1) + (y - 1) - (y - 1)(x - 1)$
23. $Q(x, y) = x^2(y - \pi)$
25. $\Delta z = 0.23, dz = 0.22$
27. $\Delta z = 0.04196, dz = 0.0414$

Section 2-6:

1. $18t^{17}$
3. 0
5. 0
7. $2 \cos(2t) - \sin(2t)$
9. -1
11. $\frac{\partial w}{\partial u} = 4u^3v^2 + 6(u + v)^5, \frac{\partial w}{\partial v} = 2u^4v + 6(u + v)^5$
13. $\frac{\partial w}{\partial u} = 0, \frac{\partial w}{\partial v} = 0$
15. $w_x = \frac{-x}{t^2} e^{-x^2/2t} - \frac{2x}{t^3} e^{-x^2/t}, w_t = -\frac{1}{t^2} e^{-x^2/(2t)} + \frac{1}{2t^3} x^2 e^{-x^2/(2t)} - \frac{2}{t^3} e^{-x^2/t} + \frac{1}{t^4} x^2 e^{-x^2/t}$
17. $\sin(t^3) + \int_0^t u^2 \cos(u^2 t) du$
19. 0

21. $\nabla f = \langle 2x, 3y^2 \rangle$
 $df/dt = 4t^3 + 9t^8$
23. $\nabla f = \langle 2x + 2y, 2x \rangle$
 $df/dt = -\sin 2t + 2 \cos 2t$
25. $\nabla f = \langle \cos(y), -x \sin(y) \rangle$
 $df/dt = \cos(2t)$
27. $\nabla f = \langle -\exp(-x^2), \exp(-y^2) \rangle$
 $= e^{-\sin^2 t} \cos t + e^{-\cos^2 t} \sin t$

Section 2-7:

1. $D_{\mathbf{i}}f = e^x \sin(y) : (\sin y) e^x$
3. $D_{\mathbf{u}}f = -2x - y^5$
5. $(D_{\mathbf{u}}f)(1, 1) = \frac{9}{5}\sqrt{5}$
7. $(D_{\mathbf{v}}f)\left(\frac{\pi}{2}, 0\right) = 0$
9. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, (D_{\mathbf{u}}f)(1, 1) = \sqrt{2}$
11. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, (D_{\mathbf{u}}f) = 3\sqrt{2}$
13. \mathbf{u} not defined, $\nabla f\left(\frac{\pi}{2}, 0\right) = \langle 0, 0 \rangle$
15. $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, (D_{\mathbf{u}}f) = \sqrt{2}$

Section 2-8:

1. min at $(0, 0)$
3. *saddle* at $(-2, 1)$
5. *saddle* at $(2, 1)$
7. min at $\left(\frac{13}{12}, \frac{-3}{4}\right)$
9. min at $(2, 0)$, saddle at $(0, 0)$
11. *saddle* at $(0, 0)$, max at $(-1, -1)$
13. *saddle* at $(0, 0)$, min at $(1, 1)$ and $(-1, -1)$
15. *saddles* at $(0, n\pi)$ for any integer n
17. *saddle* at $(0, 0)$
19. min at $(-1, 0)$, max at $(1, 0)$
21. $y = x$
23. $y = 74 + 2.4x$
25. $y = 6.15 - .5x$
27. $y = 37.15 - 0.01x$
29. $m = \frac{y_2 - y_1}{x_2 - x_1}, b = \frac{x_2y_1 - x_1y_2}{x_2 - x_1}$

Section 2-9:

1. min at $(-\frac{3}{13}\sqrt{13}, -\frac{2}{13}\sqrt{13})$
max at $(\frac{3}{13}\sqrt{13}, \frac{2}{13}\sqrt{13})$
3. min at $(-\sqrt{5}, 2\sqrt{5})$
max at $(\sqrt{5}, -2\sqrt{5})$
5. min at $(1, 0), (-1, 0)$
max at $(0, 1), (0, -1)$
7. min at $(\frac{\pm 1}{\sqrt{2}}, \frac{\pm 1}{\sqrt{2}}), (\frac{\pm 1}{\sqrt{2}}, \frac{\mp 1}{\sqrt{2}})$
max at $(1, 0)$
9. min at $(0, 1), (0, -1)$
max at $(1, 0), (-1, 0)$

Find the point(s) on the given curve closest to the origin.

11. $(\frac{1}{2}, \frac{1}{2})$
13. $(1, 1), (-1, -1)$
15. $(-0.84629, 0.39388)$