1. Express the solution of this system as a translation of a vector space:

\begin{align*}
2x_1 + x_2 - x_4 &= 5 \\
-5x_1 - 3x_2 - x_3 + x_4 &= -7 \\
5x_1 + 4x_2 + 3x_3 + 2x_4 &= 6 \\
x_1 + x_2 + x_3 + x_4 &= 1
\end{align*}

(A1, A2, A3, A7, B4)

2. Find the solution set of \( A\vec{x} = \vec{b} \) where

\[ A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -12 & 5 \\ 2 & -9 & -1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 8 \\ 26 \\ 14 \end{bmatrix}. \]

(A1, A7, B4)

3. Consider the plane in \( \mathbb{R}^3 \) which contains the vectors \([1, 2, 3]\) and \([4, 5, 6]\) and passes through the point \((7, 8, 9)\). Find the equation of the plane (in terms of \(x, y, \) and \(z\) coordinates) and express the plane as a translation of a vector space. \((B4, B12)\)

4. Consider the inner product space \( C_{-\pi, \pi} \) of continuous functions on \([-\pi, \pi]\) with the inner product of \(f\) and \(g\) defined as

\[ \langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x) \, dx. \]

In the vector space \( C_{-\pi, \pi} \), find the angle between \(\cos x\) and \(\sin x\). \((B8, B9, C15)\)
5. Show that \( P_2 \), the vector space of all polynomials of degree 2 or less, is isomorphic to \( \mathbb{R}^3 \) (actually construct the isomorphism and verify that it is an isomorphism). \( (C12, C13, C15) \)

6. Consider the vectors \( \vec{v}_1 = x^2 + 2x + 3 \), \( \vec{v}_2 = 7x^2 - 5x + 2 \), and \( \vec{v}_3 = -4x^2 + 2x - 9 \) in \( P_2 \), the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain. \( (C5, C11, C15) \)

7. Find the orthogonal complement of \( \text{span}\{[-1, 2, 0, 3], [0, 4, 1, -2]\} \) in \( \mathbb{R}^4 \). \( (C3, C18) \)

8. Transform the basis \( \{(1, 0, 1), (0, 1, 2), (2, 1, 0)\} \) for \( \mathbb{R}^3 \) into an orthogonal basis using the Gram-Schmidt process. \( (C17, C19, C20, C21) \)

9. Find the eigenvalues (they are integers) and the eigenvectors of:

\[
A = \begin{bmatrix}
-2 & 0 & 0 \\
-5 & -2 & -5 \\
5 & 0 & 3
\end{bmatrix}.
\]

\( (A9, D14, D17, D18, D19) \)

10. Consider

\[
A = \begin{bmatrix}
2 & 2 & 0 & 4 \\
3 & 3 & 2 & 2 \\
0 & 1 & 3 & 2 \\
2 & 0 & 2 & 1
\end{bmatrix}.
\]

Put \( A \) in a row echelon form, keeping track of how each row operation affects the determinant. Then calculate the determinant of \( A \). \( (A4, D15) \)