Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side**. You may omit two problems. Indicate which two problems you are omitting: _____ and ____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

   $A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$

   and express the solution as a translation of a vector space. (A1, A7, B4)

2. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = \vec{0}$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. (A9, C7)

3. Find the projection of $[1, 2, 3, 4]$ onto the plane $x + 2y + z - w = 0$. Explain your reasoning! (B3, B7, B8, C17, C19)

4. State the definition of vector space. (C1)

5. Let $T : P_2 \to P_3$, where $P_3$ is the vector space of all polynomials of degree 3 or less, be defined by $T((p(x)) = D(p(x))$, the derivative of $p(x)$. Let the ordered basis for $P_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix $A$ which represents $T$ relative to $B, B'$. (C7, C8, C11, C15)

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (C17, C19, C20, C21)
7. Express $A$ and $A^{-1}$ as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$  

(D3, D7, D9)

8. Prove that if $\lambda$ is an eigenvalue of an $n \times n$ matrix $A$, then the set $E_\lambda$ consisting of the zero vector together with all eigenvectors of $A$ for this eigenvalue $\lambda$ is a subspace of $n$-space. (C4, D17, D19)

9. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate $A^{100}$. Of course, you may leave your answer in terms of powers of certain numbers. (D1, D17, D20)

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$  

Explain your reasoning. (D23)