LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2014b, Prepared by Dr. Robert Gardner

December 5, 2014

NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Consider the matrix $A = \begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix}$. Put A in (1) row echelon form, and (2) reduced row echelon form. At each step, tell the elementary row operation you are using. (**A3, A4, A5**)

2. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = \vec{0}$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. **(A9, C7)**

3. Find the projection of $[1, 2, 1, 2]$ onto the plane $x + y + z + w = 0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. **(C5, C11, C15)**

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\| \|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Use the properties of determinants to calculate the determinant of

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 6 & 2 & 1 & 4 \\ 6 & 3 & 9 & 12 \\ 2 & 1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D12, D14, D15)**

9. Diagonalize $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$. (**D2, D17, D18, D19, D20**)

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D23)**