# LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2006, Prepared by Dr. Robert Gardner

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NAME________________________ STUDENT NUMBER__________________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit. **No calculators!**

**1.** Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. **(A1, A7, B4)**

**2.** Let $A$ be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

**3.** Consider the inner product space $C_{-\pi,\pi}$ of continuous functions on $[-\pi, \pi]$ with the inner product of $f$ and $g$ defined as

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x))g(x)\, dx.$$

In the vector space $C_{-\pi,\pi}$ find the angle between $\cos x$ and $\sin x$. **(B8, B9, C15)**

**4.** State the definition of *vector space*. **(C1)**

**5.** Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain. **(C5, C11, C15)**

**6.** Let $T : \mathcal{P}_3 \to \mathcal{P}_3$ be defined by $T(p(x)) = D(p(x))$, the derivative of $p(x)$. Let the ordered basis for $\mathcal{P}_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix $A$ which represents $T$ relative to $B, B'$. **(C7, C8, C11, C15)**

**7.** Consider

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. **(A4, A5, D6, D10)**

**8.** Find the orthogonal complement of $\text{span}\{[-1, 2, 0, 3], [0, 4, 1, -2]\}$ in $\mathbb{R}^4$. **(C3, C18)**

**9.** Find the eigenvalues (they are integers) and the eigenvectors of:

$$A = \begin{bmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{bmatrix}.$$

**(A9, D14, D17, D18, D19)**

**10.** Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate $A^{100}$. **(D1, D17, D20)**