1. Find the solution set of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -12 & 5 \\ 2 & -9 & -1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 8 \\ 26 \\ 14 \end{bmatrix}.$$  

(A1, A7, B4)

2. Give three conditions on $n \times n$ matrix $A$ which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (A5, A8, A9)

3. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C_{0,2\pi}$ of continuous functions on $[0, 2\pi]$ with the inner product of $f$ and $g$ defined as

$$\langle f, g \rangle = \int_0^{2\pi} f(x)g(x) \, dx.$$  

(B8, B9, C15)

4. Find the projection of vector $[1, 2, 3, 4]$ onto the line joining the points $(0, 4, -3, 2)$ and $(1, 4, 0, 2)$. (B3, B7, B8, B17)

5. Show that $P_2$, the vector space of all polynomials of degree 2 or less, is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism). (C12, C13, C15)

6. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1, x_1 + x_2 + x_3)$. (C7, C8)

7. Consider the space $P_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis $(x, x^2 - 1, x^3, 2x^2)$. (C11, C6, A1)

8. Subspaces $U$ and $W$ of $\mathbb{R}^n$ are orthogonal if $\vec{u} \cdot \vec{w} = 0$ for all $\vec{u} \in U$ and all $\vec{w} \in W$. Let $U$ and $W$ be orthogonal subspaces of $\mathbb{R}^n$, and let $\dim(U) = n - \dim(W)$. Show that each subspace is the orthogonal complement of the other. (C18)
9. Transform the basis \{(1, 0, 1), (0, 1, 2), (2, 1, 0)\} for \(\mathbb{R}^3\) into an orthogonal basis using the Gram-Schmidt process. (C17, C19, C20, C21)

10. Prove that for \(A = [a_{ij}]\) and \(B = [b_{ij}]\) \(n \times n\) matrices, we have \((AB)^T = B^T A^T\). (D1, D4)

11. Prove that if \(\lambda\) is an eigenvalue of an \(n \times n\) matrix \(A\), then the set \(E_\lambda\) consisting of the zero vector together with all eigenvectors of \(A\) for this eigenvalue \(\lambda\) is a subspace of \(n\)-space. (C4, D17, D19)

12. Diagonalize \(A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}\) and calculate \(A^{100}\). (D1, D17, D20)