LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2019a, Prepared by Dr. Robert Gardner

November 22, 2019

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit two problems. Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Find the solution set of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (**A1, A7, B4**)

2. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = \vec{0}$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. **(A9, C7)**

3. Explain the difference between a *vector* in $\mathbb{R}^n$ and a *point* in $\mathbb{R}^n$. (**B1, B3**)

4. Find the projection of $[1, 2, 1, 2]$ onto the plane $x+y+z+w = 0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

5. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C_{0,2\pi}$ of continuous functions on $[0, 2\pi]$with the inner product of f and g defined as

$$\langle f, g\rangle = \int_0^{2\pi} f(x)g(x)\,dx.$$

(**B8, B9, C15**)

6. Consider the space $\mathcal{P}_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis$(x, x^2 - 1, x^3, 2x^2)$. (**C11, C6, A1**).

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle\vec{v}, \vec{w}\rangle| \leq \|\vec{v}\|\|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1, 2, 0, 2],$ $[2, 1, 1, 1]$, $[1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. (**C4, D17, D19**)

10. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. What are the eigenspaces? **(D12, D17, D18, D19)**