Be clear and give all details. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. No calculators and turn off your cell phones! You may omit two problems. Indicate which two problems you are omitting: ____ and _____. There is a three hour time limit.

1. Find the solution set of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (A1, A7, B4)

2. Give three conditions on $n \times n$ matrix $A$ which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (A5, A8, A9)

3. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C_{0,2\pi}$ of continuous functions on $[0, 2\pi]$ with the inner product of $f$ and $g$ defined as

$$\langle f, g \rangle = \int_0^{2\pi} f(x)g(x) \, dx.$$  

(B8, B9, C15)

4. State the definition of vector space. (C1)

5. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1, x_1 + x_2 + x_3)$. (C7, C8)

6. Consider the space $P_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis $(x, x^2 - 1, x^3, 2x^2)$. (C11, C6, A1)

7. Transform the basis $\{(1,0,1), (0,1,2), (2,1,0)\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. (C17, C19, C20, C21)
8. Express $A$ and $A^{-1}$ as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}. $$

(D3, D7, D8, D9)

9. Find the eigenvalues (they are integers) and the eigenvectors of (A9, D14, D17, D18, D19):

$$A = \begin{bmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{bmatrix}. $$

10. Consider

$$A = \begin{bmatrix} 2 & 2 & 0 & 4 \\ 3 & 3 & 2 & 2 \\ 0 & 1 & 3 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}. $$

Put $A$ in a row echelon form, keeping track of how each row operation affects the determinant, then calculate the determinant of $A$. (A4, D15)