LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2016b, Prepared by Dr. Robert Gardner

November 18, 2016

NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution set of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (**A1, A7, B4**)

2. Give three conditions on $n \times n$ matrix A which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (**A5, A8, A9**)

3. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C_{0,2\pi}$ of continuous functions on $[0, 2\pi]$with the inner product of f and g defined as

$$\langle f, g \rangle = \int_0^{2\pi} f(x)g(x)\, dx.$$

(**B8, B9, C15**)

4. State the definition of *vector space*. **(C1)**

5. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1, x_1 + x_2 + x_3)$. (**C7, C8**)

6. Consider the space $\mathcal{P}_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis$(x, x^2 - 1, x^3, 2x^2)$. (**C11, C6, A1**).

7. Transform the basis $\{(1, 0, 1), (0, 1, 2), (2, 1, 0)\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. (**C17, C19, C20, C21**)

8. Express A and A^{-1} as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$

(**D3, D7, D8, D9**)

9. Find the eigenvalues (they are integers) and the eigenvectors of (**A9, D14, D17, D18, D19**):

$$A = \begin{bmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{bmatrix}.$$

10. Consider

$$A = \begin{bmatrix} 2 & 2 & 0 & 4 \\ 3 & 3 & 2 & 2 \\ 0 & 1 & 3 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}.$$

Put A in a row echelon form, keeping track of how each row operation affects the determinant, then calculate the determinant of A. **(A4, D15)**