LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2003, Prepared by Dr. Robert Gardner

November 21, 2003

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit two numbered problems. Indicate which two problems you are omitting: _____ and _____. NO CALCULATORS!!! Time limit: 3 hours.

1. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix}.$$

Put A in (1) row echelon form, and (2) reduced row echelon form. At each step, tell the elementary row operation you are using. **(A3, A4, A5)**

2. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = \vec{0}$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. **(A9, C7)**

3. Find the projection of $[1, 2, 3, 4]$ onto the plane $x + y + z + w = 0$. **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Find a basis for $\text{span}\{1, 4x + 3, 3x - 4, x^2 + 2, x - x^2\}$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. **(A3, A4, C5, C6, C11, C15)**

6. Transform the basis $\{[1, 1, 0], [0, 1, 2], [1, 1, 1]\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. **(C17, C19, C20, C21)**

7. If A and B are $n \times n$ matrices and if A is singular, prove that AB is also singular. **(D1, D8, D14)**

8. Find the rank, nullity, and a basis for the column space of **(A3, A4, A5, D6, D10)**:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & -4 & 2 \\ 3 & -10 & -7 \end{bmatrix}.$$

9. Diagonalize $A = \begin{bmatrix} -3 & 5 \\ -2 & 4 \end{bmatrix}$ and calculate A^{100}. **(D1, D17, D20)**

10. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**