LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2004, Prepared by Dr. Robert Gardner

October 8, 2004

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix}.$$

Put A in (1) row echelon form, and (2) reduced row echelon form. At each step, tell the elementary row operation you are using. **(A3, A4, A5)**

2. Let A be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

3. Consider the plane in $\mathbb{R}^3$ which contains the three points $(1,0,0)$, $(0,1,-1)$, and $(1,1,1)$. Find the equation of the plane (in terms of x, y, and z coordinates) and express the plane as a translation of a vector space. **(B4, B12)**

4. State the definition of *vector space*. **(C1)**

5. Show that $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less, is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism and verify that it is an isomorphism). **(C12, C13, C15)**

6. Transform the basis $\{[1,1,0],[0,1,2],[1,1,1]\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. **(C17, C19, C20, C21)**

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\|\|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Find the rank, nullity, and a basis for the column space of **(A9, D14, D17, D18, D19)**:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & -4 & 2 \\ 3 & -10 & -7 \end{bmatrix}.$$

9. Find the eigenvalues (they are integers) and the eigenvectors of **(A9, D14, D17, D18, D19)**:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ -4 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}.$$

10. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**