NAME __________________________ Start Time: _______ End Time: _______

Be clear and give all details. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. No calculators and turn off your cell phones! Use the paper provided and only write on one side. You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (A1, A7, B4)

2. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = 0$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. (A9, C7)

3. Find the projection of $[1, 2, 3, 4]$ onto the plane $x + 2y + z - w = 0$. Explain your reasoning! (B3, B7, B8, C17, C19)

4. State the definition of vector space. (C1)

5. Let $T : \mathcal{P}_2 \to \mathcal{P}_3$, where $\mathcal{P}_3$ is the vector space of all polynomials of degree 3 or less, be defined by $T((p(x)) = D(p(x))$, the derivative of $p(x)$. Let the ordered basis for $\mathcal{P}_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix $A$ which represents $T$ relative to $B, B'$. (C7, C8, C11, C15)

6. Transform the basis $\{[1, 1, 1], [1, 0, 1], [0, 1, 1]\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. (C17, C19, C20, C21)

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq ||\vec{v}||||\vec{w}||$, to prove the Triangle Inequality in an inner-product space. (B8, B10, C15)
8. If $A$ is an invertible $n \times n$ matrix, then $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$. Use this fact to find $A^{-1}$ for

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix}.$$  

(D13, D15, D16)

9. Prove that if $\lambda$ is an eigenvalue of an $n \times n$ matrix $A$, then the set $E_{\lambda}$ consisting of the zero vector together with all eigenvectors of $A$ for this eigenvalue $\lambda$ is a subspace of $n$-space. (C4, D17, D19)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate $A^{100}$. Of course, you may leave your answer in terms of powers of certain numbers. (D1, D17, D20)