Be clear and give all details. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide.

No calculators and turn off your cell phones! You may omit two problems. Indicate which two problems you are omitting: _____ and ____. There is a three hour time limit.

1. Find the solution of \( \mathbf{A} \mathbf{x} = \mathbf{b} \) where
   \[
   \mathbf{A} = \begin{bmatrix}
   1 & -4 & 1 \\
   3 & -13 & 0 \\
   2 & -9 & -1
   \end{bmatrix}
   \quad \text{and} \quad
   \mathbf{b} = \begin{bmatrix}
   -2 \\
   -10 \\
   -8
   \end{bmatrix}
   \]
   and express the solution as a translation of a vector space. (A1, A7, B4)

2. Explain the difference between a vector in \( \mathbb{R}^n \) and a point in \( \mathbb{R}^n \). (B1, B3)

3. Find the projection of \([1, 2, 1, 2]\) onto the plane \(x + y + z + w = 0\). Explain your reasoning! (B3, B7, B8, C17, C19)

4. State the definition of vector space. (C1)

5. Consider the vectors \( \mathbf{v}_1 = x^2 - 2x + 1, \mathbf{v}_2 = 2x^2 + 5x + 11, \) and \( \mathbf{v}_3 = 3x^2 + 7x + 17 \) in \( \mathcal{P}_2 \), the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. (C5, C11, C15)

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace \( \text{span}\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\} \) of \( \mathbb{R}^4 \). (C17, C19, C20, C21)

7. Use the Schwarz Inequality, which states that for vectors \( \mathbf{v} \) and \( \mathbf{w} \) in an inner-product space, we have \(|\langle \mathbf{v}, \mathbf{w} \rangle| \leq \|\mathbf{v}\||\mathbf{w}\|\), to prove the Triangle Inequality in an inner-product space. (B8, B10, C15)
8. Consider

\[ A = \begin{bmatrix}
0 & 6 & 6 & 3 \\
1 & 2 & 1 & 1 \\
4 & 1 & -3 & 4 \\
1 & 3 & 2 & 0
\end{bmatrix}. \]

Find the rank, a basis for the row space, and a basis for the column space. (A4, A5, D6, D10)

9. Prove that if \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A \), then the set \( E_\lambda \) consisting of the zero vector together with all eigenvectors of \( A \) for this eigenvalue \( \lambda \) is a subspace of \( n \)-space. (C4, D17, D19)

10. Diagonalize \( A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix} \) and calculate \( A^{100} \). Of course, you may leave your answer in terms of powers of certain numbers. (D1, D17, D20)