LINEAR ALGEBRA COMPREHENSIVE EXAM

Fall 2003, Prepared by Dr. Robert Gardner

September 25, 2003

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit two numbered problems. Indicate which two problems you are omitting: _____ and _____. NO CALCULATORS!!! Time limit: 3 hours.

1. Express the solution of this system as a translation of a vector space:

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 4 \\ -3x_1 & + & 2x_2 & & & + & x_4 & = & 0 \\ x_1 & + & x_2 & + & 5x_3 & - & 4x_4 & = & 3 \\ & & 5x_2 & + & 11x_3 & - & 6x_4 & = & 10 \end{array}$$

   (A1, A2, A3, A7, B4)

2. Find the projection of $[1, 2, 3, 4]$ onto the plane $x + y + z + w = 0$. **(B3, B7, B8, C17, C19)**

3. Explain the difference between a vector in $\mathbb{R}^n$ and a point in $\mathbb{R}^n$. **(B3)** What is a vector space isomorphism? **(C13)**

4. State the definition of *vector space*. **(C1)**

5. Let $T : \mathcal{P}_3 \to \mathcal{P}_3$ be defined by $T(p(x)) = D(p(x))$, the derivative of $p(x)$. Let the ordered basis for $\mathcal{P}_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix A which represents T relative to B, B'. **(C7, C8, C11, C15)**

6. Transform the basis $\{[1, 1, 0], [0, 1, 2], [1, 1, 1]\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. **(C17, C19, C20, C21)**

7. Prove that for $A = [a_{ij}]$ and $B = [b_{ij}]$ $n \times n$ matrices, we have $(AB)^T = B^T A^T$. **(D1, D4)**

8. Find A^{-1} and express it as a product of elementary matrices where

$$A = \begin{bmatrix} 14 & -8 & -1 \\ -17 & 10 & 1 \\ -19 & 11 & 1 \end{bmatrix}.$$

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**

10. Find the rank, basis for the row space, and basis for the column space for matrix A where

$$A = \begin{bmatrix} 2 & 2 & 0 & 4 \\ 3 & 3 & 2 & 2 \\ 1 & 1 & -2 & 6 \\ 4 & 4 & 4 & 0 \end{bmatrix}.$$