1. Solve the system

\[
\begin{align*}
x_1 & - 2x_2 + x_3 - x_4 = 4 \\
2x_1 & - 3x_2 + 2x_3 - x_4 = -1 \\
3x_1 & - 5x_2 + 3x_3 - 4x_4 = 3 \\
-x_1 & + x_2 - x_3 + 2x_4 = 5
\end{align*}
\]

\((A1, A2, A3, A6, A7)\)

2. Prove that if \(\vec{x}_1\) and \(\vec{x}_2\) are both solutions to the homogeneous system of equations \(A\vec{x} = \vec{0}\), then any linear combination of \(\vec{x}_1\) and \(\vec{x}_2\) is also a solution. \((A9)\)

3. Find the coordinate vector of \(2x^2 + 3x - 1\) relative to the ordered basis \((x^2 - x, 2x^2 - 2x + 1, x^2 - 2x)\). \((A1, C6, C11)\)

4. Find a unit vector in \(\mathbb{R}^3\) which is orthogonal to both \(\vec{x} = (1, 2, 3)\) and \(\vec{y} = (0, 1, 1)\). \((B8, B9, B12)\)

5. Find the standard matrix representation (i.e. the representation with respect to the standard bases of \(\mathbb{R}^m\) and \(\mathbb{R}^n\)) of the linear transformation \(T : \mathbb{R}^3 \rightarrow \mathbb{R}^2\) defined by \(T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, x_1)\). \((C7, C8)\)

6. State the definition of vector space. \((C1)\)

7. Use Schwarz’s Inequality, which states that for vectors \(\vec{v}\) and \(\vec{w}\) in an inner-product space we have \(|⟨\vec{v}, \vec{w}⟩| \leq ||\vec{v}|| ||\vec{w}||\), to prove the Triangle Inequality in an inner-product space. \((B8, B10, C15)\)

8. Let \(A\) be an \(n \times n\) matrix. Prove that the collection of all solutions to the equation \(A\vec{x} = \vec{0}\) form a subspace of \(\mathbb{R}^n\) \((A9, C2, C4)\)

9. Express \(A\) and \(A^{-1}\) as products of elementary matrices where

\[
A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.
\]

\((D3, D7, D8, D9)\)
10. Consider

\[ A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}. \]

Find the rank, a basis for the row space, and a basis for the column space.

11. Prove that if \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A \), then the set \( E_\lambda \) consisting of the zero vector together with all eigenvectors of \( A \) for this eigenvalue \( \lambda \) is a subspace of \( n \)-space. (C4, D17, D19)

12. Consider

\[ A = \begin{bmatrix} 2 & 2 & 0 & 4 \\ 3 & 3 & 2 & 2 \\ 0 & 1 & 3 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}. \]

Put \( A \) in a row echelon form, keeping track of how each row operation affects the determinant. Then calculate the determinant of \( A \). (A4, D15)