LINEAR ALGEBRA COMPREHENSIVE EXAM

Summer 2016, Prepared by Dr. Robert Gardner

August 5, 2016

NAME _____ Start Time: _____ End Time: _____

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. **(A1, A7, B4)**

2. Let A be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

3. Consider the inner product space $C_{-\pi,\pi}$ of continuous functions on $[-\pi, \pi]$ with the inner product of f and g defined as

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x))g(x)\,dx.$$

In the vector space $C_{-\pi,\pi}$ find the angle between $\cos x$ and $\sin x$. **(B8, B9, C15)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. **(C5, C11, C15)**

6. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1, x_1 + x_2 + x_3)$. **(C7, C8)**

7. Express A and A^{-1} as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$

(**D3, D7, D8, D9**)

8. Transform the basis $\{(1,0,1), (0,1,2), (2,1,0)\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. (**C17, C19, C20, C21**)

9. Find two of the eigenvalues (they are integers) and two of the eigenvectors for (**A9, D14, D17, D18, D19**):

$$A = \begin{bmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{bmatrix}.$$

10. Consider

$$A = \begin{bmatrix} 2 & 2 & 0 & 4 \\ 3 & 3 & 2 & 2 \\ 0 & 1 & 3 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}.$$

Put A in a row echelon form, keeping track of how each row operation affects the determinant. Then calculate the determinant of A. (**A4, D15**)