LINEAR ALGEBRA COMPREHENSIVE EXAM

Summer 2014, Prepared by Dr. Robert Gardner

August 1, 2014

NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. **(A1, A7, B4)**

2. Explain the difference between a *vector* in $\mathbb{R}^n$ and a *point* in $\mathbb{R}^n$. (**B1, B3**)

3. Find the projection of $[1, 2, 1, 2]$ onto the plane $x + y + z + w = 0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. **(C5, C11, C15)**

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\| \|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Consider

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. (**A4, A5, D6, D10**)

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. (**C4, D17, D19**)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate A^{100}. Of course, you may leave your answer in terms of powers of certain numbers. (**D1, D17, D20**)