NAME ___________________________ STUDENT NUMBER ___________________________

Be clear and give all details. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide.

No calculators! You may omit two problems. Indicate which two problems you are omitting: ___ and ___. There is a three hour time limit.

1. Consider the matrix

\[
A = \begin{bmatrix}
0 & 2 & -1 & 3 \\
-1 & 1 & 2 & 0 \\
1 & 1 & -3 & 3 \\
1 & 5 & 5 & 9
\end{bmatrix}.
\]

Put \(A\) in (1) row echelon form, and (2) reduced row echelon form. At each step, tell the elementary row operation you are using. (A3, A4, A5)

2. Let \(A\) be an \(m \times n\) matrix. Show that \(\{\vec{x} \mid A\vec{x} = \vec{0}\}\) is a subspace of \(\mathbb{R}^n\). (A8, A9, C4)

3. Consider the plane in \(\mathbb{R}^3\) which contains the vectors \([1, 2, 3]\) and \([4, 5, 6]\) and passes through the point \((7, 8, 9)\). Find the equation of the plane (in terms of \(x, y,\) and \(z\) coordinates) and express the plane as a translation of a vector space. (B4, B12)

4. State the definition of vector space. (C1)

5. Let \(T : \mathcal{P}_3 \rightarrow \mathcal{P}_3\) be defined by \(T(p(x)) = D(p(x))\), the derivative of \(p(x)\). Let the ordered basis for \(\mathcal{P}_3\) be \(B = B' = (x^3, x^2, x, 1)\). Find the matrix \(A\) which represents \(T\) relative to \(B, B'\). (C7, C8, C11, C15)

6. Transform the basis \([1, 1, 0], [0, 1, 2], [1, 1, 1]\) for \(\mathbb{R}^3\) into an orthogonal basis using the Gram-Schmidt process. (C17, C19, C20, C21)

7. Use the Schwarz Inequality, which states that for vectors \(\vec{v}\) and \(\vec{w}\) in an inner-product space, we have \(|\langle \vec{v}, \vec{w} \rangle| \leq ||\vec{v}|| ||\vec{w}||\), to prove the Triangle Inequality in an inner-product space. (B8, B10, C15)
8. Consider

\[
A = \begin{bmatrix}
0 & 6 & 6 & 3 \\
1 & 2 & 1 & 1 \\
4 & 1 & -3 & 4 \\
1 & 3 & 2 & 0
\end{bmatrix}.
\]

Find the rank, a basis for the row space, and a basis for the column space. \((A_4, A_5, D_6, D_{10})\)

9. Find the eigenvalues (they are integers) and the eigenvectors of \((A_9, D_{14}, D_{17}, D_{18}, D_{19})\):

\[
A = \begin{bmatrix}
-2 & 0 & 0 \\
-5 & -2 & -5 \\
5 & 0 & 3
\end{bmatrix}.
\]

10. What does it mean for matrix \(A\) to be an **orthogonal matrix**? Prove that for \(n \times n\) orthogonal matrix \(A\), we have \(\|Ax\| = \|-Ax\|\). \((D_{21})\)