LINEAR ALGEBRA COMPREHENSIVE EXAM

Summer 2018, Prepared by Dr. Robert Gardner

July 20, 2018

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Show that if the system of equations $A\vec{x} = \vec{b}$ has two distinct solutions, then it has an infinite number of solutions. **(A1, A6, A8)**

2. Let A be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

3. Find the projection of $[1,2,1,2]$ onto the plane $x+y+z+w=0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and **ordered bases**. If you are not using ordered bases then you are not arguing correctly!!! **(C5, C11, C15)**

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1,2,0,2], [2,1,1,1], [1,0,1,1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w}\rangle| \le \|\vec{v}\|\|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Consider

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. (**A4, A5, D6, D10**)

9. Show that the vector space

$$V = \{m + a\cos(x) + b\sin(x) \mid m, a, b \in \mathbb{R}\}$$

is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism). (**C12, C13, C15**)

10. Find the eigenvalues (they are integers) and <u>all</u> the eigenvectors of (**A9, D14, D17, D18, D19**):

$$A = \begin{bmatrix} -2 & 0 & 0 \\ -5 & -2 & -5 \\ 5 & 0 & 3 \end{bmatrix}.$$