LINEAR ALGEBRA COMPREHENSIVE EXAM

Summer 2015, Prepared by Dr. Robert Gardner

July 24, 2015

NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Prove that a linear system of equations with two different solutions has an infinite number of solutions. **(A1, A6, C7)**

2. Consider the plane in $\mathbb{R}^3$ which contains the vectors $[1,2,3]$ and $[4,5,6]$ and passes through the point $(7,8,9)$. Find the equation of the plane (in terms of x, y and z coordinates) and express the plane as a translation of a vector space. (**B4, B12**)

3. Find the projection of $[1,2,1,2]$ onto the plane $x+y+z+w=0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. Give an example of a vector space other than $\mathbb{R}^n$ or $\mathbb{C}^n$. **(C1)**

5. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1,x_2,x_3)) = (x_1+x_2, x_2+x_3, x_3+x_1, x_1+x_2+x_3)$. (**C7, C8**)

6. Consider the vectors $\vec{v}_1 = x^2+2x+3$, $\vec{v}_2 = 7x^2-5x+2$, and $\vec{v}_3 = -4x^2+2x-9$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain. **(C5, C11, C15)**

7. Find the determinant of

$$A = \begin{bmatrix} 2 & 0 & -1 & 7 \\ 6 & 1 & 0 & 4 \\ 8 & -2 & 1 & 0 \\ 4 & 1 & 0 & 2 \end{bmatrix}$$

by first row reducing the first two columns and keeping track of how the row reduction affects the determinant of A. **(D12, D15)**

8. (a) What is an elementary matrix? **(D7)**

(b) Express A and A^{-1} as products of elementary matrices where $A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$. **(D3, D7, D8, D9)**

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**

10. Diagonalize

$$A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$$

and find A^5. **(D1, D17, D18, D20, D22)**