1. Express the solution of this system as a translation of a vector space:

\[
\begin{align*}
3x_1 + 2x_2 + x_3 &= 5 \\
-5x_1 - 3x_2 - x_3 + x_4 &= -7 \\
5x_1 + 4x_2 + 3x_3 + 2x_4 &= 6 \\
x_1 + x_2 + x_3 + x_4 &= 1
\end{align*}
\]

(A1, A2, A3, A7, B4)

2. Consider the matrix

\[
A = \begin{bmatrix}
1 & 14 & -4 & 7 \\
-3 & -6 & 0 & -9 \\
5 & -8 & 6 & 9 \\
2 & 13 & -3 & 9
\end{bmatrix}
\]

Which columns contain pivots? What is a basis for the column space? (A3, A4, A5, D6)

3. Give three conditions on \( n \times n \) matrix \( A \) which would (each) imply that the system \( Ax = b \) has a unique solution. Give two conditions which would (each) imply that \( Ax = b \) has multiple solutions. (A5, A8, A9)

4. Find the projection of vector \([1, 2, 3, 4]\) onto the line joining the points \((0, 4, -3, 2)\) and \((1, 4, 0, 2)\). (B3, B7, B8, C17)

5. Use the Schwarz Inequality, which states that for vectors \( \vec{v} \) and \( \vec{w} \) in an inner-product space, we have \( |\langle \vec{v}, \vec{w} \rangle| \leq ||\vec{v}||||\vec{w}|| \), to prove the Triangle Inequality in an inner-product space. (B8, B10, C15)
6. Consider the vectors \( \vec{v}_1 = x^2 + 2x + 3 \), \( \vec{v}_2 = 7x^2 - 5x + 2 \), and \( \vec{v}_3 = -4x^2 + 2x - 9 \) in \( P_2 \), the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain. (C5, C11, C15)

7. Find the orthogonal complement of \( \text{span}\{[-1, 2, 0, 3], [0, 4, 1, -2]\} \) in \( \mathbb{R}^4 \). (C3, C18)

8. Consider

\[
A = \begin{bmatrix}
2 & 2 & 0 & 4 \\
3 & 3 & 2 & 2 \\
0 & 1 & 3 & 2 \\
2 & 0 & 2 & 1
\end{bmatrix}.
\]

Put \( A \) in a row echelon form, keeping track of how each row operation affects the determinant. Then calculate the determinant of \( A \). (A4, D15)

9. Prove that if \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A \), then the set \( E_\lambda \) consisting of the zero vector together with all eigenvectors of \( A \) for this eigenvalue \( \lambda \) is a subspace of \( n \)-space. (C4, D17, D19)

10. Diagonalize \( A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix} \) and calculate \( A^{100} \). (D1, D17, D20)