LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2015b, Prepared by Dr. Robert Gardner

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NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. State three conditions on $n \times n$ matrix A which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Does the system

$$\begin{bmatrix} 2 & 4 & -2 \\ 4 & 8 & 3 \\ -1 & -3 & 0 \end{bmatrix} \vec{x} = \vec{0}$$

have a unique solution (explain)? (**A5, A8, A9**)

2. Consider the plane in $\mathbb{R}^3$ which contains the vectors $[1,2,3]$ and $[4,5,6]$ and passes through the point $(7,8,9)$. Find the equation of the plane (in terms of x, y and z coordinates) and express the plane as a translation of a vector space. (**B4, B12**)

3. Find the projection of $[1,2,1,2]$ onto the plane $x+y+z+w=0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. Give an example of a vector space other than $\mathbb{R}^n$ or $\mathbb{C}^n$. **(C1)**

5. Transform the basis $\{[1,1,1],[1,0,1],[0,1,1]\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. **(C17, C19, C20, C21)**

6. Consider the vectors $\vec{v}_1 = x^2+2x+3$, $\vec{v}_2 = 7x^2-5x+2$, and $\vec{v}_3 = -4x^2+2x-9$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain. **(C5, C11, C15)**

7. Consider

$$\begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. **(A4, A5, D6, D10)**

8. (a) What is an elementary matrix? **(D7)**

(b) Express A and A^{-1} as products of elementary matrices where $A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$. **(D3, D7, D8, D9)**

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D23)**