LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2014b, Prepared by Dr. Robert Gardner

May 2, 2014

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. **(A1, A7, B4)**

2. Use Schwarz's Inequality, which states that

$$|\vec{v}_1 \cdot \vec{v}_2| \leq \|\vec{v}_1\| \, \|\vec{v}_2\| \text{ for all } \vec{v}_1, \vec{v}_2 \in \mathbb{R}^n$$

to prove the Triangle Inequality in $\mathbb{R}^n$. (**B8, B10**)

3. Find the projection of $[1, 2, 1, 2]$ onto the plane $x + y + z + w = 0$. Explain your reasoning! **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. **(C5, C11, C15)**

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace $\text{span}\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

7. Show that the vector space

$$V = \{m + a\cos(x) + b\sin(x) \mid m, a, b \in \mathbb{R}\}$$

is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism). (**C12, C13, C15**)

8. Consider

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. (**A4, A5, D6, D10**)

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. (**C4, D17, D19**)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate A^{100}. Of course, you may leave your answer in terms of powers of certain numbers. (**D1, D17, D20**)