LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2017b, Prepared by Dr. Robert Gardner

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NAME __________________________ STUDENT NUMBER ____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

To address potential academic misconduct during the test, I will wander the room and may request to see the progress of your work on the test while you are taking it. You are not allowed to access your phone during the test. You are not allowed to stop during a test to go to the bathroom, unless you have presented a documented medical need beforehand.

You may omit two problems. Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Express the solution of this system as a translation of a vector space:

$$\begin{array}{rcrcrcrcr} 3x_1 & + & 2x_2 & + & x_3 & & & = & 4 \\ -5x_1 & - & 3x_2 & - & x_3 & + & x_4 & = & -7 \\ 5x_1 & + & 4x_2 & + & 3x_3 & + & 2x_4 & = & 6 \\ x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \end{array}$$

(A1, A2, A3, A7, B4)

2. Let A be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

3. Find the part of vector $[-2, 4, 1, 0]$ which is perpendicular to plane $x + y + z + w = 5$. **(B3, B7, B8, C17, C19)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain *using ordered bases*! **(C5, C11, C15)**

6. Let $T : \mathcal{P}_3 \to \mathcal{P}_3$ be defined by $T(p(x)) = D(p(x))$, the derivative of $p(x)$. Let the ordered basis for $\mathcal{P}_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix A which represents T relative to B, B'. **(C7, C8, C11, C15)**

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\|\|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Express A and A^{-1} as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$

(**D3, D7, D8, D9**)

9. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate A^{100}. **(D1, D17, D20)**

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D23)**