LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2019a, Prepared by Dr. Robert Gardner

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NAME________________________ STUDENT NUMBER____________________________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit two problems. Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Express the solution of this system as a translation of a vector space:

$$\begin{array}{rcrcrcrcr} 3x_1 & + & 2x_2 & + & x_3 & & & = & 4 \\ -5x_1 & - & 3x_2 & - & x_3 & + & x_4 & = & -7 \\ 5x_1 & + & 4x_2 & + & 3x_3 & + & 2x_4 & = & 6 \\ x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \end{array}$$

(A1, A2, A3, A7, B4)

2. Let A be an $m \times n$ matrix. Show that $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$. **(A8, A9, C4)**

3. Find the projection of vector $[1, 2, 3, 4]$ onto the line joining the points $(0, 4, -3, 2)$ and $(1, 4, 0, 2)$. **(B3, B7, B8, C17)**

4. State the definition of *vector space*. **(C1)**

5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain *using ordered bases*! **(C5, C11, C15)**

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span$\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of $\mathbb{R}^4$. (**C17, C19, C20, C21**)

7. Show that the vector space

$$V = \{m + a\cos(x) + b\sin(x) \mid m, a, b \in \mathbb{R}\}$$

is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism). (**C12, C13, C15**)

8. Express A and A^{-1} as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$

(**D3, D7, D8, D9**)

9. Let A and C be matrices such that the product AC is defined. Prove that the column space of AC is contained in the column space of A. (**D6, D10**)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate A^{100}. **(D1, D17, D20)**