1. Express the solution of this system as a translation of a vector space:

\[
\begin{align*}
3x_1 + 2x_2 + x_3 &= 4 \\
-5x_1 - 3x_2 - x_3 + x_4 &= -7 \\
5x_1 + 4x_2 + 3x_3 + 2x_4 &= 6 \\
x_1 + x_2 + x_3 + x_4 &= 1
\end{align*}
\]

(A1, A2, A3, A7, B4)

2. Let \( A \) be an \( m \times n \) matrix. Show that \( \{ \vec{x} \mid A\vec{x} = \vec{0} \} \) is a subspace of \( \mathbb{R}^n \). (A8, A9, C4)

3. Find the projection of vector \([1, 2, 3, 4]\) onto the line joining the points \((0, 4, -3, 2)\) and \((1, 4, 0, 2)\). (B3, B7, B8, C17)

4. State the definition of vector space. (C1)

5. Consider the vectors \( \vec{v}_1 = x^2 - 2x + 1 \), \( \vec{v}_2 = 2x^2 + 5x + 11 \), and \( \vec{v}_3 = 3x^2 + 7x + 17 \) in \( \mathcal{P}_2 \), the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using ordered bases! (C5, C11, C15)

6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace \( \text{span}\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\} \) of \( \mathbb{R}^4 \). (C17, C19, C20, C21)

7. Show that the vector space

\[ V = \{ m + a \cos(x) + b \sin(x) \mid m, a, b \in \mathbb{R} \} \]

is isomorphic to \( \mathbb{R}^3 \) (actually construct the isomorphism). (C12, C13, C15)
8. Express $A$ and $A^{-1}$ as products of elementary matrices where

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 4 \end{bmatrix}.$$  

(D3, D7, D8, D9)

9. Let $A$ and $C$ be matrices such that the product $AC$ is defined. Prove that the column space of $AC$ is contained in the column space of $A$. (D6, D10)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate $A^{100}$. (D1, D17, D20)