LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2015a, Prepared by Dr. Robert Gardner

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NAME ______________________________ Start Time: ________ End Time: ________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Consider the matrix $A = \begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix}$. Put A in (1) row echelon form, and (2) reduced row echelon form. At each step, tell the elementary row operation you are using. (**A3, A4, A5**)

2. Solve the system of equations (**A2, A6, A7**):

$$\begin{array}{rcrcrcr} & & x_2 & - & 3x_3 & = & -5 \\ 2x_1 & + & 3x_2 & - & x_3 & = & 7 \\ 4x_1 & + & 5x_2 & - & 2x_3 & = & 10 \end{array}$$

3. Prove that if $\vec{x}_1$ and $\vec{x}_2$ are both solutions to the homogeneous system of equations $A\vec{x} = \vec{0}$, then any linear combination of $\vec{x}_1$ and $\vec{x}_2$ is also a solution. **(A9, C7)**

4. State the definition of *vector space*. **(C1)**

5. Let $T : \mathcal{P}_3 \to \mathcal{P}_3$ be defined by $T(p(x))$, the derivative of $p(x)$. let the ordered basis for $\mathcal{P}_3$ be $B = B' = (x^3, x^2, x, 1)$. Find the matrix A which represents T relative to B, B'. **(C7, C8, C11, C15)**

6. Find the projection of x onto $\sin x$ in the inner product space $C_{0,\pi}$ of continuous functions on $[0, \pi]$ with the inner product of f and g defined as **(B8, C15, C17)**:

$$\langle f, g \rangle = \int_0^\pi f(x)g(x)\,dx.$$

7. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \le \|\vec{v}\|\|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

8. Use row reduction and the properties of determinants to calculate the determinant of

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 6 & 2 & 1 & 4 \\ 6 & 3 & 9 & 12 \\ 2 & 1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D12, D14, D15)**

9. Diagonalize $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$. (**D2, D17, D18, D19, D20**)

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}.$$

Explain your reasoning. **(D23)**