LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2013, Prepared by Dr. Robert Gardner February 1, 2013

NAME ______________________________ Start time: _________ End time: _________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The bold faced numbers in parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!!!** You may omit two numbered problems. Indicate which two problems you are omitting: ______ and ______.

1. Prove that a linear system of equations with two different solutions has an infinite number of solutions. **(A1, A6, C7)**

2. Express solutions of the system

$$\begin{array}{rcrcrcrcl} x_1 & - & 2x_2 & + & x_3 & - & x_4 & = & 0 \\ 2x_1 & - & 3x_2 & + & 2x_3 & - & 3x_4 & = & 0 \\ 3x_1 & - & 5x_2 & + & 3x_3 & - & 4x_4 & = & 0 \\ -x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & 0 \end{array}$$

   as the translation of a vector space. **(A1, A2, A3, A7, A9, B4)**

3. Give three conditions on $n \times n$ matrix A which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (**A5, A8, A9**)

4. Use the Schwarz Inequality, which states that for vectors $\vec{v}$ and $\vec{w}$ in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\| \|\vec{w}\|$ to prove the Triangle Inequality in an inner-product space. **(B8, B10, C15)**

5. Find the projection of vector $[1, 2, 1, 2]$ onto the plane $x + y + z + w = 0$. **(B3, B7, B8, C17, C19)**

6. State the definition of *vector space*. **(C1)**

7. Consider the space $\mathcal{P}_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis $(x, x^2 - 1, x^3, 2x^2)$. **(A1, C6, C11)**.

8. Transform the basis $\{(1, 0, 1), (0, 1, 2), (2, 1, 0)\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. **(C17, C19, C20, C21)**

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A, then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n-space. **(C4, D17, D19)**

10. Diagonalize $\begin{bmatrix} -3 & 5 \\ -2 & 4 \end{bmatrix}$ and find A^{100}. **(D1, D17, D18, D20, D22)**