1. Find the solution set of $A\vec{x} = \vec{b}$ where
\[
A = \begin{bmatrix}
1 & -4 & 1 \\
3 & -13 & 0 \\
2 & -9 & -1
\end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix}
-2 \\
-10 \\
-8
\end{bmatrix}
\]
and express the solution as a translation of a vector space. (A1, A7, B4)

2. Give three conditions on $n \times n$ matrix $A$ which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (A5, A8, A9)

3. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C_{0,2\pi}$ of continuous functions on $[0, 2\pi]$ with the inner product of $f$ and $g$ defined as
\[
\langle f, g \rangle = \int_0^{2\pi} f(x)g(x) \, dx.
\]
(B8, B9, C15)

4. Find the projection of vector $[1, 2, 3, 4]$ onto the line joining the points $(0, 4, -3, 2)$ and $(1, 4, 0, 2)$. (B3, B7, B8, C17)

5. Find the standard matrix representation (i.e. the representation with respect to the standard bases of $\mathbb{R}^m$ and $\mathbb{R}^n$) of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1, x_1 + x_2 + x_3)$. (C7, C8)

6. Consider the space $P_3$ of all polynomials of degree 3 or less. Find the coordinate vector of $x^3 + 3x^2 - 4x + 2$ relative to the ordered basis $(x, x^2 - 1, x^3, 2x^2)$. (C11, C6, A1)

7. Transform the basis $\{(1, 0, 1), (0, 1, 2), (2, 1, 0)\}$ for $\mathbb{R}^3$ into an orthogonal basis using the Gram-Schmidt process. (C17, C19, C20, C21)

8. Prove that for $A = [a_{ij}]$ and $B = [b_{ij}]$ $n \times n$ matrices, we have $(AB)^T = B^T A^T$. (D1, D4)
9. Prove that if \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A \), then the set \( E_\lambda \) consisting of the zero vector together with all eigenvectors of \( A \) for this eigenvalue \( \lambda \) is a subspace of \( n \)-space. (C4, D17, D19)

10. Diagonalize \( A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix} \) and calculate \( A^{100} \). (D1, D17, D20)