1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (A1, A7, B4)

2. Give three conditions on an $n \times n$ matrix $A$ which would (each) imply that the system $A\vec{x} = \vec{b}$ has a unique solution. Give two conditions which would (each) imply that $A\vec{x} = \vec{b}$ has multiple solutions. (A5, A8, A9)

3. Show that the vectors $\sin x$ and $\cos x$ are orthogonal in the inner product space $C([0, 2\pi])$ of continuous functions on $[0, 2\pi]$ with inner product of $f$ and $g$ defined as

$$\langle f, g \rangle = \int_{0}^{2\pi} f(x)g(x) \, dx.$$  

(B8, B9, C15)

4. State the definition of vector space. (C1)

5. Consider the vectors $\vec{v}_1 = x^2 + 2x + 3$, $\vec{v}_2 = 7x^2 - 5x + 2$, and $\vec{v}_3 = -4x^2 + 2x - 9$ in $\mathcal{P}_2$, the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using ordered bases. (C5, C11, C15)

6. Find the orthogonal complement of span$\{[-1, 2, 0, 3], [0, 4, 1, -2]\}$ in $\mathbb{R}^4$. (C3, C18)
7. Show that $P_2$, the vector space of all polynomials of degree 2 or less, is isomorphic to $\mathbb{R}^3$ (actually construct the isomorphism and verify that it is an isomorphism). (C12, C13, C15)

8. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\} of $\mathbb{R}^4$. (C17, C19, C20, C21)

9. Find the eigenvalues (they are integers) and the corresponding eigenvectors of:

$$A = \begin{bmatrix}
-2 & 0 & 0 \\
-5 & -2 & -5 \\
5 & 0 & 3
\end{bmatrix}.$$

(A3, D14, D17, D18, D19)

10. Find the L/U decomposition of the matrix

$$A = \begin{bmatrix}
1 & 3 & -1 \\
2 & 8 & 4 \\
-1 & 3 & 4
\end{bmatrix}.$$

Explain your reasoning. (D23)