

LINEAR ALGEBRA COMPREHENSIVE EXAM

Spring 2014a, Prepared by Dr. Robert Gardner

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NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit two problems. Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Find the solution of $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -4 & 1 \\ 3 & -13 & 0 \\ 2 & -9 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ -10 \\ -8 \end{bmatrix}$$

and express the solution as a translation of a vector space. (**A1, A7, B4**)

2. Explain the difference between a *vector* in \mathbb{R}^n and a *point* in \mathbb{R}^n . (**B1, B3**)
3. Find the projection of $[1, 2, 1, 2]$ onto the plane $x + y + z + w = 0$. Explain your reasoning! (**B3, B7, B8, C17, C19**)
4. State the definition of *vector space*. (**C1**)
5. Consider the vectors $\vec{v}_1 = x^2 - 2x + 1$, $\vec{v}_2 = 2x^2 + 5x + 11$, and $\vec{v}_3 = 3x^2 + 7x + 17$ in \mathcal{P}_2 , the vector space of all polynomials of degree 2 or less. Are these vectors linearly independent? Explain using linear algebra and ordered bases. (**C5, C11, C15**)
6. Use the Gram-Schmidt process to find an orthonormal basis for the subspace $\text{span}\{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]\}$ of \mathbb{R}^4 . (**C17, C19, C20, C21**)
7. Use the Schwarz Inequality, which states that for vectors \vec{v} and \vec{w} in an inner-product space, we have $|\langle \vec{v}, \vec{w} \rangle| \leq \|\vec{v}\| \|\vec{w}\|$, to prove the Triangle Inequality in an inner-product space. (**B8, B10, C15**)

8. Consider

$$A = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}.$$

Find the rank, a basis for the row space, and a basis for the column space. (**A4, A5, D6, D10**)

9. Prove that if λ is an eigenvalue of an $n \times n$ matrix A , then the set E_λ consisting of the zero vector together with all eigenvectors of A for this eigenvalue λ is a subspace of n -space. (**C4, D17, D19**)

10. Diagonalize $A = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$ and calculate A^{100} . Of course, you may leave your answer in terms of powers of certain numbers. (**D1, D17, D20**)