1. Express the solution of this system as a translation of a vector space:

\[
\begin{align*}
3x_1 + 2x_2 + x_3 &= 5 \\
-5x_1 - 3x_2 - x_3 + x_4 &= -7 \\
5x_1 + 4x_2 + 3x_3 + 2x_4 &= 6 \\
x_1 + x_2 + x_3 + x_4 &= 1
\end{align*}
\]

(A1, A2, A3, A7, B4)

2. Consider the matrix

\[
A = \begin{bmatrix}
1 & 14 & -4 & 7 \\
-3 & -6 & 0 & -9 \\
5 & -8 & 6 & 9 \\
2 & 13 & -3 & 9
\end{bmatrix}.
\]

Which columns contain pivots? What is a basis for the column space? (A3, A4, A5, D6)

3. Consider the plane in \(\mathbb{R}^3\) which contains the vectors \([1, 2, 3]\) and \([4, 5, 6]\) and passes through the point \((7, 8, 9)\). Find the equation of the plane (in terms of \(x, y,\) and \(z\) coordinates) and express the plane as a translation of a vector space. (B4, B12)

4. State the “Fundamental Theorem of Finite Dimensional Vector Spaces” (which deals with isomorphisms of vector spaces). Let \(V_1\) and \(V_2\) be vector spaces with real scalars. What conditions must be satisfied for \(\pi : V_1 \rightarrow V_2\) to be an isomorphism? Are the vector spaces \(\mathbb{R}^n\) and \(\mathbb{C}^n\) isomorphic? Explain. (B4, B12)

5. Let \(A\) be an \(n \times n\) matrix. Prove that the collection of all solutions to the equation \(A\vec{x} = \vec{0}\) form a subspace of \(\mathbb{R}^n\). (A9, C2, C4)
6. Consider the vectors \( \vec{v}_1 = x^2 + 2x + 3, \) \( \vec{v}_2 = 7x^2 - 5x + 2, \) and \( \vec{v}_3 = -4x^2 + 2x - 9 \) in \( P_2, \) the vector space of all polynomials of degree 2 or less. Are these vectors linearly dependent? Explain. \( \text{(C5, C11, C15)} \)

7. Use the Gram-Schmidt process to find an orthonormal basis for the subspace span{[1, 2, 0, 2], [2, 1, 1, 1], [1, 0, 1, 1]} of \( \mathbb{R}^4. \) \( \text{(C17, C19, C20, C21)} \)

8. (a) What is an elementary matrix? \( \text{(D7)} \)
   
   (b) Express \( A \) and \( A^{-1} \) as a product of elementary matrices where \( A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}. \) \( \text{(D3, D7, D8, D9)} \)

9. Prove that if \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A, \) then the set \( E_\lambda \) consisting of the zero vector together with all eigenvectors of \( A \) for this eigenvalue \( \lambda \) is a subspace of \( n \)-space. \( \text{(C4, D17, D19)} \)

10. Find the eigenvalues and eigenvectors of \( A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \) What is the eigenspace? \( \text{(D12, D17, D18, D19)} \)