STUDY GUIDE FOR LINEAR ALGEBRA COMPREHENSIVE EXAM

Department of Mathematics, M.S. Program
Prepared by Dr. Robert Gardner
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Statement of Purpose: The purpose of the comprehensive linear algebra exam is to force students to recall material from sophomore linear algebra. By reviewing this material, it is expected that the math graduate school experience will be enhanced. This will prepare students for graduate classes such as Real Analysis (and the study of Banach and Hilbert spaces), Applied Math (and the concept of eigenspaces and linearization), numerical linear algebra, and, of course, matrix theory. No calculators will be allowed during the exam.

When tests are given: The linear algebra comprehensive exam will be offered twice each Fall and Spring semester. The test will be given either during the week before classes start or in the first two weeks of classes, and sometime shortly after the middle of each term. Assuming demand, the test will also be offered once over the summer. You will choose 8 questions from a list of 10 and have 3 hours to complete the test.

Material possible covered: Questions on the test will be drawn from the following topics. Expect both theoretical and computational questions.

A. Linear Systems of Equations

A1. Systems of linear equations and solutions.

A2. Augmented matrices.

A3. Elementary row operations.

A4. Row echelon form and reduced row echelon form.

A5. Pivots.

A6. Consistent and inconsistent systems.

A7. Gaussian reduction/Gauss-Jordan method.

A8. Conditions of A for $A\vec{x} = \vec{b}$ to have a solution.

A9. Homogeneous systems and their solutions.

B. Introductory Vector Concepts

B1. Euclidean space $\mathbb{R}^n$.

B2. Vector addition and subtraction.

B3. The difference between a vector in $\mathbb{R}^n$ and a point in $\mathbb{R}^n$.

B4. Translation of vectors.

B5. Scalars and scalar multiplication.

B6. Distribution, associativity, and commutivity properties of scalar multiplication and vector addition.

B7. Parallel vectors and the direction of a vector in $\mathbb{R}^n$.

B8. Dot products and norms.

B9. The angle between vectors and orthogonal vectors.

B10. Schwarz's Inequality and the Triangle Inequality.

B11. Commutivity and distributive properties of dot products.

B12. Cross Product and properties.

C. Vector Spaces

C1. Definition of Vector Space.

C2. Linear combination of vectors.

C3. Span of vectors.

C4. Subspaces of a given space.

C5. Linear dependence and independence.

C6. Basis and dimension.

C7. Linear transformations.

C8. Matrix representation of linear transformations.

C9. Composition of linear transformations.

C10. Invertible transformations.

C11. Ordered basis and coordinate vector relative to an ordered basis.

C12. One-to-one transformation and onto transformation.

C13. Vector space isomorphism.

C14. The "Fundamental Theorem of Finite Dimensional Vector Spaces" (namely, that any [real] n-dimensional vector space is isomorphic to $\mathbb{R}^n$).

C15. Inner product spaces and examples other than $\mathbb{R}^n$.

C16. Generation of a norm using the inner product.

C17. Projections of one vector in the direction of another.

C18. Orthogonal complement of a subspace of a given space.

C19. Projection of a vector onto a subspace.

C20. Orthogonal and orthonormal bases.

C22. Gram-Schmidt process to generate orthogonal basis from a given basis.

Matrices

D1. Matrix products and sums.

D2. Diagonal matrices.

D3. Identity matrix.

D4. Transpose of a matrix.

D5. Symmetric matrix.

D6. Row and column space of a matrix.

D7. Elementary matrices and the representation of elementary row operations by multiplication by elementary matrices.

D8. Invertible matrices and singular matrices.

D9. Computation of inverse matrices.

D10. Rank and the Rank Equation ($\text{rank}(A)+\text{nullity}(A) = \#$ columns of A).

D11. Conditions for A to be invertible.

D12. Determinant of $n \times n$ matrix (defined recursively) and their computations.

D13. Cofactor and minor matrices.

D14. Properties of determinants.

D15. Computation of determinant and the ways determinants are affected by elementary row operations.

D16. Adjoint of a matrix A and its relationship to A^{-1}.

D17. Eigenvalues and eigenvectors.

D18. Characteristic polynomial.

D19. Eigenspace of an eigenvalue.

D20. Diagonalizable matrix and the computation of C and D for a given A such that $A = CDC^{-1}$.

D21. Orthogonal matrices and their properties.

D22. Similar matrices.

D23. L/U decomposition of matrices.