CALCULUS COMPREHENSIVE EXAM

Fall 2010, Prepared by Dr. Jeff Knisley

December 16, 2010

NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 which contains Calculus 1 material) and one problem from numbers 6 through 10 (which contains Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. Do each of the following:

   (a) State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).

   (b) Use the definition of to prove that $\lim_{x\to a} mx + b = ma + b$. **(1)**

2. What does it mean for a function $y = g(x)$ to be *implicit* to an equation $F(x, y) = 0$? For the equation $x^2 + y^2 = 1$, find y' and find 3 functions implicit to this equation. **(10)**

3. State the Extreme Value Theorem for Continuous Functions. Consider $f(x) = x^{2/3}$ on the interval $[-2, 3]$. Find the absolute extrema of f on this interval. **(8, 12, 16)**

4. The intensity of illumination at any point is proportional to the product of the strength of the light source and the inverse of the square of the distance from the source. If two sources of relative strengths a and b are a distance c apart, at what point on the line joining them will the intensity be minimum? Assume the intensity at any point is the sum of intensities from the two sources. **(16, 18, 34)**

5. State both parts of the Fundamental Theorem of Calculus. Evaluate $\displaystyle\int_1^2 \frac{(\ln x)^7}{x}\,dx$ and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computation. **(23, 24, 31)**

6. The region bounded by the positive x-axis, the positive y-axis, and $y = e^{-x}$ is revolved about the y-axis. What's the volume? **(26, 31, 38)**

7. State L'Hôpital's Rule for an ∞/∞ indeterminate form. Use L'Hôpital's Rule to show $\displaystyle\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$. **(31, 37)**

8. Evaluate $\displaystyle\int_{-1}^{1} x^{-4/3}\,dx$. WARNING: Be careful — $f(x) = x^{-4/3}$ is not continuous on $[-1, 1]$. **(23, 39)**

9. State the Integral Test (which concerns the convergence of a positive term series). Show that for $p > 1$, the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. **(38, 43)**

10. Do each of the following **(46)**:

    (a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge?).

    (b) What is the radius of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}$ (give detailed reasons for your answer).