CALCULUS COMPREHENSIVE EXAM

Fall 2010, Prepared by Dr. Jeff Knisley
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NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. Do each of the following:

   (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

   (b) Prove that if $\lim_{x\to a} f(x) = L$ and $k \neq 0$, then

   $$\lim_{x\to a} \left(\frac{f(x)}{k}\right) = \frac{L}{k} \qquad (\mathbf{2})$$

2. Do each of the following:

   (a) State the definition of *derivative* of a function f. **(6)**

   (b) Use the **definition** of the derivative to find $f'(x)$ given that $f(x) = \dfrac{1}{\sqrt{x}}$. **(2, 6, 8)**

3. Consider $f(x) = \ln\left|x^2 - 1\right|$ (note the absolute values). Find the first and second derivative of f, find where f is increasing/decreasing, find where f is concave up/concave down, find the asymptotes of the graph of f, find the extrema of f, and graph $y = f(x)$. (**8, 14, 15, 16, 17**)

4. Find the point(s) (x, y) on the curve $y = 4.5 - x^2$ closest to the origin **(12, 16, 18)**.

5. Do each of the following **(23, 24)**:

   (a) State the two parts of the Fundamental Theorem of Calculus.

   (b) Use the Fundamental Theorem of Calculus to evaluate $\displaystyle\int_0^1 x \sin x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem.

6. (a) Use the definition of $y = \sin^{-1} x$ (in terms of the sine function) and implicit differentiation to find $y' = \dfrac{d}{dx}\left[\sin^{-1} x\right]$.

   (b) Evaluate $\displaystyle\int_0^{3\sqrt{2}/4} \frac{dx}{\sqrt{9 - 4x^2}}$. (**28, 34, 35**)

7. Evaluate **(37, 38, 39)**:

   (a) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$.

   (b) $\int_0^{\infty}\frac{e^x}{1+e^{2x}}\,dx$ (use all notation correctly and don't write things that don't make sense).

8. Find the volume of the solid generated by revolving about the x-axis the region in the first quadrant enclosed by the coordinate axes, the curve $y = 2/(1+x^2)$ and the line $x = 1$. **(24, 26)**

9. Do each of the following **(46)**:

   (a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge?).

   (b) What is the interval of convergence for $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n+1}$. Is it open, closed, or neither? Explain.

10. Do each of the following **(44, 47)**:

    (a) Use the geometric series to find the MacLaurin series of $\ln(1+x)$.

    (b) Use (a) to derive a series representation for $\ln(2)$. Does the resulting series converge absolutely?