CALCULUS COMPREHENSIVE EXAM

Fall 2009, Prepared by Dr. Jeff Knisley

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NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. (a) State the definition of the limit of a function (that is, what does $\lim_{x \to a} f(x) = L$ mean?).
   (b) Use the definition to prove that $\lim_{x \to 0} 2x + 3 = 3$. **(1)**

2. (a) State the definition of a function $f(x)$ being continous at a point p. **(4)**
   (b) Prove that if $f(x)$ is differentiable at a point p, then $f(x)$ is also continuous at a point p. **(7)**

3. (a) State the product rule.
   (b) Evaluate the derivative of $f(x) = x^2 e^x \sin(x)$ and indicate with a (*) where you are applying the product rule **(8, 31, 34)**

4. Using the second derivative test, show that $f(x) = 2x + e^{-x}$ has an absolute minimum on $\mathbb{R}$. What is that minimum and at what point is it located? **(15, 16, 18, 31)**.

5. (a) State both parts of the Fundamental Theorem of Calculus.
   (b) Calculate $F'(x)$ and simplify completely given that $F(x) = \int_0^{\ln(x)} \sin\left(e^t\right) dt$. Indicate with a star $(*)$ where you have used the Fundamental Theorem of Calculus in your computation. **(20, 23, 34)**

6. What is the volume of the solid obtained by revolving $y = \cos(x)$, x in $[0, \pi]$, about the y-axis. ? **(23, 24, 26, 34)**

7. Convert to a limit and evaluate $\int_0^{\infty} \frac{e^x}{e^{2x} + 1} dx$. **(20, 24, 35, 38)**

8. (a) State the definition of the limit of a sequence $\lim_{n\to\infty} a_n = L$. **(41)**

   (b) Use the definition to prove that $\lim_{n\to\infty} \frac{\cos(n\pi)}{n\pi} = 0$. (**28, 31, 45**).

9. (a) Use the definition of the sum of a series to prove that if $\sum_{n=1}^{\infty} |a_n| = 0$, then $a_n = 0$ for all n. **(41)**

   (b) Determine if the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n[\ln(n)]^2}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use. (**28, 31, 45**).

10. Find the Taylors series expansion of $f(x) = \sin(x)$ at $x = \pi$ (show your work). Show and/or explain why the series converges absolutely for all x. (**31,45,46,47**)