CALCULUS COMPREHENSIVE EXAM

Fall 2009, Prepared by Dr. Jeff Knisley

October 23, 2009

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. (a) State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).
   (b) Use the definition to prove that $\lim_{x\to 1} 2x+3 = 5$. **(1)**

2. State the Sandwich Theorem and use it to prove that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$. **(3)**

3. (a) State the Chain Rule.
   (b) Evaluate the derivative of $f(x) = e^{2x}\sin(e^{2x})$ and indicate with a (*) where you are applying the chain rule **(8, 31, 34)**

4. Using the second derivative test, show that $f(x) = 2x + e^{-x}$ has an absolute minimum on $\mathbb{R}$. What is that minimum and at what value of x is it located? **(15, 16, 18, 31)**.

5. (a) State both parts of the Fundamental Theorem of Calculus.
   (b) Calculate $F'(x)$ and simplify completely given that $F(x) = \int_0^{\tan(x)} \frac{1}{\sqrt{t^2+1}}dt$. Indicate with a star ($*$) where you have used the Fundamental Theorem of Calculus in your computation. **(20, 23, 34)**

6. What is the length of the curve $y = 2x^{3/2}$ *for* x *in* $[0,1]$? **(23, 24, 27)**

7. The hyperbolic tangent and hyperbolic secant are, respectively,

$$\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1} \quad \textit{and} \quad \operatorname{sech}(x) = \frac{2e^{x}}{e^{2x}+1}$$

Show that $\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$. What does the result imply about the monotonicity of $\tanh(x)$? (**8, 14, 31**)

8. Use a trigonometric substitution to evaluate the indefinite integral $\displaystyle\int \frac{x^2}{x^2+1}dx$. (other methods can be used to check your work, but a trig substitution computation is required) **(20, 24, 28)**

9. (a) State the definition of the sum of a series $\displaystyle\sum_{n=1}^{\infty} a_n = L$. **(41)**

   (b) Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use. (**28, 31, 45**).

10. Find a MacLaurin Series for $f(x) = e^{2x} - e^{x}$ (show your work). Show and/or explain why the series converges absolutely for all x. Use the series to calculate $\displaystyle\lim_{x\to 0} \frac{e^{2x} - e^{x}}{x}$ (**31,45,46,47**)