CALCULUS COMPREHENSIVE EXAM

Fall 2005, Prepared by Dr. Robert Gardner

December 2, 2005

NAME ________________________ STUDENT NUMBER ____________________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Do each of the following **(1)**:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Use the definition of limit to prove that $\lim_{x\to -2} -3x + 1 = 7$.

2. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(c) Find $\dfrac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$.

3. Do each of the following **(12, 16, 18)**:

(a) State the Extreme Value Theorem.

(b) Find the extrema of $f(x) = x^4 - 8x^2$ for $x \in [-1, 3]$.

4. A metal rod has the shape of a right circular cylinder. As it is being heated, its length is increasing at a rate of 0.005 cm/min and its diameter is increasing at 0.002 cm/min. At what rate is the volume changing when the rod has length 40 centimeters and diameter 3 centimeters?

5. Do each of the following **(21 23)**:

(a) State the definition of *partition*, *norm* of a partition, *Reimann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.

(b) State the two parts of the Fundamental Theorem of Calculus.

6. Do each of the following **(29)**:

(a) State the definition of $\ln x$ (using integrals).

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$.

7. Do each of the following **(31, 37, 39)**:

(a) Evaluate $\lim_{x\to 0^+}\left(1+\frac{1}{x}\right)^x$.

(b) Evaluate $\int_{-\infty}^{\infty}\frac{1}{x^2}\,dx$.

8. Do each of the following **(41)**:

(a) State the definition of the limit of a sequence: $\lim_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Evaluate $\sum_{n=1}^{\infty}\left(1-\frac{1}{2^n}\right)$.

9. Do each of the following **(44, 45, 46)**:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) Consider $\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n^2+3}}$. Find the interval of convergence, the radius of convergence, and the values for which the convergence is absolute or conditional.

10. Find a Maclaurin Series for $f(x) = e^x$ (show your work). Where does the series converge absolutely? Where does it converge conditionally? Where does it diverge? Use the series to verify that $\int e^x\,dx = e^x + C$. **(31, 45, 46, 47)**