CALCULUS COMPREHENSIVE EXAM

Fall 2012, Prepared by Dr. Robert Gardner

November 30, 2012

NAME ________________________ Start time: _________ End time: _________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit. No calculators (this is a *math* test)!

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

(b) Prove that if $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a}(kf(x)) = kL$ **(1,2)**

2. Do each of the following **(8, 10, 31, 34)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(c) Differentiate (you need not simplify your answer): $f(x) = \ln\sqrt{\dfrac{\cot(e^x)}{\arctan(x)}}$.

3. Car A leaves an intersection going due north at a rate of 50 miles per hour. At the same time, Car B heads west from the same intersection going 40 miles per hour. How fast is the distance between the cars increasing two hours after leaving the intersection? **(19)**

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 xe^x dx$ and indicate with a star (*) where you are applying the Fundamental Theorem. **(23, 24)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. **(a)** Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\tan^{-1} x]$.

(b) Evaluate $\displaystyle\int \frac{dx}{x^2 - 2x + 5}$. **(28, 34, 35)**

7. Do each of the following: **(38,39)**

(a) Evaluate $\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

(b) Evaluate $\int_{-1}^{1} \frac{1}{x^2}\,dx$.

8. Do each of the following:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? **(45)**

9. State the Integral Test (which concerns the convergence of a positive term series). Show that for $p > 1$ the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. **(38, 43)**

10. Do each of the following:

(a) Determine whether $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ converges or diverges and explain. **(43)**

(b) Use the MacLaurin Series for e^x to find a series for $\int e^{-x^2}\,dx$. **(30, 46)**