CALCULUS COMPREHENSIVE EXAM

Fall 2003, Prepared by Dr. Robert Gardner

November 21, 2003

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Do each of the following:

(a) State the Sandwich Theorem (sometimes called the "Squeeze Theorem") for the limit of a function.

(b) Use the facts that $\sin\theta < \theta < \tan\theta$ for $\theta \in (0, \pi/2)$ to show that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (WARNING: This is a two sided limit and the inequality is only known to hold for $\theta \in (0, \pi/2)$.) **(3,33)**

2. Do each of the following:

(a) State the definition of *derivative* of a function f. **(6)**

(b) Use the definition to differentiate $f(x) = \frac{1}{\sqrt{x}}$. **(2, 6, 8)**

3. Do each of the following:

(a) State the Extreme Value Theorem. **(12)**

(b) Show that the largest area rectangle is in fact a square. **(12, 18)**

4. Do each of the following:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

5. Do each of the following:

(a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^{\ln 2} xe^x \, dx$ and indicate with a start $(*)$ where you are applying the Fundamental Theorem. **(24, 30)**

6. Do each of the following:

(a) State the definition of $\ln x$ (in terms of definite integrals). **(29)**

(b) Use the definition to prove that $\ln ab = \ln a + \ln b$ $(a > 0,\ b > 0)$ **(23, 29)**

7. Find the volume of the solid generated by revolving about the x-axis the region in the first quadrant enclosed by the coordinate axes, the curve $y = 2/(1 + x^2)$ and the line $x = 1$. **(24, 26)**

8. Do each of the following:

(a) Evaluate $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$. (Show all work!) **(30, 37)**

(b) Evaluate $\int_0^2 \frac{dx}{(x-1)^2}$. **(39)**

(c) Use the Integral Test to show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

9. Do each of the following:

(a) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim_{n\to\infty} a_n = L$."

(b) Let $\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\left(\sum_{n=1}^{\infty} a_n\right) = L$." **(41)**

10. Find a Maclaurin Series for $f(x) = e^x$ (show your work). Where does the series converge absolutely? Where does it converge conditionally? Where does it diverge? Use the series to verify that $\int e^x \, dx = e^x + C$. **(31, 45, 46, 47)**