CALCULUS COMPREHENSIVE EXAM

Fall 2004, Prepared by Dr. Robert Gardner

October 8, 2004

NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Do each of the following:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$. **(2)**

2. Do each of the following:

(a) State the definition of *derivative* of a function f. **(6)**

(b) Use the definition to differentiate $f(x) = \dfrac{1}{\sqrt{x}}$. **(2, 6, 8)**

3. Find the volume of the largest right circular cylinder which can be inscribed in a sphere of radius R. The volume of a right circular cylinder of radius r and height h is $V = \pi r^2 h$ and the volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$. **(12, 16, 18)**.

4. Do each of the following **(23, 24)**

(a) State the two parts of the Fundamental Theorem of Calculus. **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\displaystyle\int_0^1 x \sin x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem. **(23, 24)**

5. Do each of the following.

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\displaystyle\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 21, 23)**

6. The curve $y = \ln x$ for $x \in [1, e]$ is revolved about the y-axis to produce a water tank. How much work is done in pumping the tank full of water? The water starts at a level of $y = 0$, distances are measured in feet and the mass-density of water is 62.4 lb/ft^3. **(24, 27, 30)**

7. Consider $y = \dfrac{x^2 + 1}{e^x}$. Where is y increasing/decreasing? Where is y concave up/concave down? What are the asymptotes of y? Graph. **(14, 15, 31)**

8. Do each of the following.

(a) Evaluate $\lim\limits_{x \to 0^+} \left(1 + \dfrac{1}{x}\right)^x$. **(31, 37)**

(b) Evaluate $\displaystyle\int_{\infty}^{-\infty} \frac{2x\,dx}{(x^2+1)^2}$. **(24, 38)**

(c) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$. **(39)**

9. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim\limits_{n \to \infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Find the limit of the sequence $\{a_n\} = \{\tan^{-1} n\}$. **(35, 41)**

10. Do each of the following.

(a) For what values of x does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{x}$ converge? **(46)**

(b) Use the MacLaurin series for e^x to find a series for $\displaystyle\int e^{-x^2}\,dx$. **(47)**