CALCULUS COMPREHENSIVE EXAM

Fall 2012, Prepared by Dr. Robert Gardner

September 14, 2012

NAME _________________________ Start time: __________ End time: __________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit. No calculators (this is a *math* test)!

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. Do each of the following **(8, 10, 31, 34)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(c) Differentiate (you need not simplify your answer): $f(x) = \ln\sqrt{\dfrac{\cot(e^x)}{\arctan(x)}}$.

3. Consider $y = \dfrac{x^2 + 1}{e^x}$. Where is y increasing/decreasing? Where is y concave up/concave down? What are the asymptotes of y? Graph. **(14,15)**

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 x \sin x dx$ and indicate with a star (*) where you are applying the Fundamental Theorem. **(23, 24)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\displaystyle\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Find the length of $y = x^2$ for $x \in [0, 1]$. **(24, 27, 34)**

7. Do each of the following **(23, 29, 31)**:

(a) State the definition of $\ln(x)$ (using integrals).

(b) Use the definition from part **(a)** to prove that $\ln(ab) = \ln(a) + \ln(b)$.

8. Do each of the following: **(33,34,35)**

(a) Evaluate $\lim_{y \to 0} \frac{\sin 7y}{4y}$.

(b) Evaluate $\lim_{t \to 0^+} \left(1 + \frac{1}{t}\right)^t$.

(c) Evaluate $\int_{-1}^{1} \frac{1}{x^2}\, dx$.

9. Do each of the following:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? (**45**)

10. Do each of the following:

(a) Use the MacLaurin Series for e^x to find a series for $\int e^{-x^2} dx$. **(30, 46)**

(b) Estimate $\int_0^1 e^{-x^2} dx$ to the nearest 0.001 and explain why you know your answer has this level of accuracy. **(44, 47)**