CALCULUS COMPREHENSIVE EXAM

Fall 2008, Prepared by Dr. Robert Gardner

September 19, 2008

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$.

(c) Find $\frac{dy}{dx} : \tan^{-1}(\ln y) = e^{x^2}$.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^2 xe^x\,dx$ and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. **(a)** Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\tan^{-1} x]$.

(b) Evaluate $\displaystyle\int \frac{dx}{x^2 - 2x + 5}$. **(28, 34, 35)**

7. Do each of the following **(38, 39, 41)**:

(a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$.

8. (a) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim\limits_{n\to\infty} (a_n) = L$." **(41)**

(b) Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\displaystyle\left(\sum_{n=1}^{\infty} a_n\right) = L$." **(41)**

9. Determine whether the following series converge or diverge and explain. **(43)**

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$.

10. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?