CALCULUS COMPREHENSIVE EXAM

Fall 2006a, Prepared by Dr. Robert Gardner

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NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. (a) State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).
   (b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$. **(1,2)**

2. Prove that if f has a derivative at $x = c$, then f is continuous at $x = c$. **(4,7)**

3. (a)What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?
   (b) Find y' if $\sin(xy) = \ln(x \cot y)$. **(8, 10, 31, 34)**

4. Find the volume of the largest right circular cylinder which can be inscribed in a right circular cone of height 3 and base radius 1. The volume of a right circular cylinder of radius r and height h is $V = \pi r^2 h$ and the volume of a right circular cone of height H and base radius R is $V = \frac{1}{3}\pi R^2 H$. **(18)**

5. (a) State the Fundamental Theorem of Calculus (both parts).
   (b) Evaluate $\int_1^e \ln x\, dx$ (HINT: use parts) and indicate with a star $(*)$ where you have used the Fundamental Theorem of Calculus in your computations. **(23, 24, 31)**

6. Find the length of the curve given by the equation $y = \int_0^x \sqrt{\sec^4 t - 1}\, dt$ for $-\pi/4 \leq x \leq \pi/4$. **(23, 27)**

7. (a) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim_{n\to\infty}(a_n) = L$."
   (b) Let $\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\left(\sum_{n=1}^{\infty} a_n\right) = L$." (**41**)

8. (a) State L'Hôpital's Rule for an ∞/∞ indeterminate form.
   (b) Use L'Hôpital's Rule to show $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$. **(31, 37)**

9. Do each of the following **(38, 39, 41)**:

   (a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

   (b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$.

10. (a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

    (b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? (**46**)