CALCULUS COMPREHENSIVE EXAM

Summer 2007, Prepared by Dr. Robert Gardner

August 10, 2007

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

   (b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x)+g(x)) = L+M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(12, 18)**:

   (a) State the Extreme Value Theorem.

   (b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

   (b) Evaluate $\int_1^e \ln x \, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

   (b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Evaluate **(37, 38, 39)**:

   (a) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ (use all notation correctly and don't write things that don't make sense).

   (b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

7. State L'Hôpital's Rule for an ∞/∞ indeterminate form. Use L'Hôpital's Rule to show $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$. **(31, 37)**

8. State the definition of $g(x) = \tan^{-1} x$. Use this definition and the differentiation properties of $f(x) = \tan x$ to show that $g'(x) = \dfrac{d}{dx}[\tan^{-1} x] = \dfrac{1}{1+x^2}$. **(28, 34, 35)**

9. Do each of the following

   a. For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

   b. What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? (**46**)

10. Compute a Taylor series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\,dt$. **(47)**