CALCULUS COMPREHENSIVE EXAM

Summer 2005, Prepared by Dr. Robert Gardner

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NAME ________________________ STUDENT NUMBER ____________________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Do each of the following **(1)**:

(a) Give the definition of "the limit as x approaches a of $f(x)$ is L: $\lim_{x\to a} f(x) = L$."

(b) Use the definition to *prove* that $\lim_{x\to 4}(-3x+5) = -7$.

2. Do each of the following **(4)**:

(a,b) Suppose the interval $[a,b]$ is a subset of the domain of function f. What is the definition of "f is continuous at point c where $c \in (a,b)$"? What is the definition of "f is continuous on the interval $[a,b]$"?

(c) Use the definitions from **(a)** and **(b)** to show that $f(x) = \sqrt{1-x^2}$ is continuous on $[-1,1]$.

3. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x,y) = 0$?

(c) Find $\frac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$.

4. The intensity of illumination at any point is proportional to the product of the strength of the light source and the inverse of the square of the distance from the source. If two sources of relative strengths a and b are a distance c apart, at what point on the line joining them will the intensity be minimum? Assume the intensity at any point is the sum of intensities from the two sources.

5. Do each of the following **(21, 23)**:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.

(b) State the two parts of the Fundamental Theorem of Calculus.

6. Do each of the following **(23, 24, 35)**:

(a) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 \frac{1+x}{1+x^2}$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem..

(b) Evaluate and simplify: $\frac{d}{dx}\left[\int_0^{\tan x} \frac{dt}{\sqrt{1+t^2}}\right]$.

7. Evaluate **(37, 38, 39)**:

(a) $\lim_{x\to 0^+} x^x$.

(b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ (use all notation correctly and don't write things that don't make sense).

(c) $\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

8. Do each of the following **(39, 41, 43)**:

(a) If f is continuous on $[a,c)\bigcup(c,b]$ then state the definition of $\int_a^b f(x)\,dx$. That is, how do you integrate over a discontinuity? You may assume the usual definition for integrals of continuous functions has been established.

(b) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim_{n\to\infty} a_n = L$."

(c) Use the Integral Test to show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

9. Do each of the following **(46)**:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge).

(b) What is the radius of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}$ (give detailed reasons for your answer).

10. Do each of the following **(46, 47)**:

(a) For what values of x does $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converge conditionally, absolutely, or diverge?

(b) Use the MacLaurin series for e^x to find a series for $\int e^{-x^2}\,dx$.