CALCULUS COMPREHENSIVE EXAM

Summer 2002, Prepared by Dr. Robert Gardner

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NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 6 (which contain Calculus 1 material) and one problem from numbers 7 through 12 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).
   (b) Use the definition to prove that $\lim_{x\to 0} 4 - 2x = 4$. (**1**)

2. State the Intermediate Value Theorem. Prove that $\cos x = x$ for some x. **(5)**

3. (a) State the Chain Rule.
   (b) Suppose $F'(x) = f(x)$ for all real x. Calculate the derivative of $e^{\cos(F(\ln x))}$. **(8, 31, 34)**

4. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius R. The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$ and the volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$. **(12, 16, 18)**.

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.
   (b) Explain the difference in a definite integral and an indefinite integral (if any). (**20, 21, 23**)

6. (a) State both parts of the Fundamental Theorem of Calculus.
   (b) Evaluate $\int_0^\pi \sin x \cos x\,dx$ and indicate with a star $(*)$ where you have used the Fundamental Theorem of Calculus in your computation. **(23, 24, 34)**

7. The curve $y = \ln x$ for $x \in [1, e]$ is revolved about the y-axis to produce a water tank. How much work is done in pumping the tank full of water? The water starts at a level of $y = 0$, distances are measured in feet and the mass-density of water is 62.4 lb/ft^3. **(24, 27, 30)**

8. Consider $y = \dfrac{x^2+1}{e^x}$. Where is y increasing/decreasing? Where is y concave up/concave down? What are the asymptotes of y? Graph.

9. State the definition of $g(x) = \tan^{-1} x$. Use this definition and the differentiation properties of $f(x) = \tan x$ to show that $g'(x) = \dfrac{d}{dx}[\tan^{-1} x] = \dfrac{1}{1+x^2}$. **(28, 34, 35)**

10. (a) Evaluate $\displaystyle\int \frac{8}{(4x^2+1)^2}\,dx$. **(24, 35)**

    (b) Evaluate $\displaystyle\int_{-1}^{4} \frac{dx}{\sqrt{|x|}}$. **(38, 39)**

11. State the Integral Test (which concerns the convergence of a positive term series). Show that for $0 < p < 1$, the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. **(38, 43)**

12. Consider $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$. Find the interval of convergence, the radius of convergence, and the values for which the convergence is absolute or conditional. **(44, 45, 46)**