CALCULUS COMPREHENSIVE EXAM

Summer 2017, Prepared by Dr. Robert Gardner

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NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. Do each of the following **(1)**:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x \to a} f(x) = L$ mean?).

(b) Use the definition of limit to prove that $\lim_{x \to 8} \left(\frac{x}{2} + 5 \right) = 9$.

2. Do each of the following **(5)**:

(a) State the Intermediate Value Theorem.

(b) State the Mean Value Theorem.

(c) Prove that $\cos x = x$ for some x.

3. Do each of the following **(10)**:

(a) What does it mean for $y = f(x)$ to be a function *implicit* to the equation $F(x, y) = 0$?

(b) Find the equation of the line tangent to $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.

4. Do each of the following **(23, 24)**:

(a) State the two parts of the Fundamental Theorem of Calculus.

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 x \sin x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem.

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. (a) Use the definition of $y = \sin^{-1} x$ (in terms of the sine function) and implicit differentiation to find $y' = \dfrac{d}{dx}\left[\sin^{-1} x\right]$.

(b) Evaluate $\displaystyle\int_0^{3\sqrt{2}/4} \frac{dx}{\sqrt{9-4x^2}}$. **(28, 34, 35)**

7. Do each of the following **(31, 37, 39)**:

(a) Evaluate $\displaystyle\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x$.

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

8. Do each of the following **(39, 41, 43)**:

(a) If f is continuous on $[0,\infty)$, then state the definition of $\displaystyle\int_0^{\infty} f(x)\,dx$. You may assume the usual definition for integrals of continuous functions on closed and bounded intervals has been established.

(b) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim_{n\to\infty} a_n = L$."

(c) Use the Integral Test to show that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

9. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?

10. Compute the Taylor series for $\ln x$ centered at $a = 1$. What is the radius of convergence? **(31, 46, 47)**