CALCULUS COMPREHENSIVE EXAM

Summer 2009, Prepared by Dr. Robert Beeler and Dr. Robert Gardner

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NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(12, 18)**:

(a) State the Extreme Value Theorem.

(b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^e \ln x \, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. **(a)** Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\tan^{-1} x]$.

(b) Evaluate $\displaystyle\int \frac{dx}{x^2 - 2x + 5}$. **(28, 34, 35)**

7. **(a)** Use the definition of $y = \sin^{-1} x$ (in terms of the sine function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\sin^{-1} x]$. **(35)**

(b) Evaluate $\displaystyle\int \frac{dx}{\sqrt{5 - 4x - x^2}}$. **(28, 34, 35)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim\limits_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} ne^{-2n}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use. **(45)**

9. Do each of the following:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) For what values of x does $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-5)^n}{5^n(n+5)}$ converge? **(46)**

10. Find a Maclaurin Series for $f(x) = e^x$ (show your work). Where does the series converge absolutely? Where does it converge conditionally? Where does it diverge? Use the series to verify that $\int e^x dx = e^x + C$. **(31, 45, 46, 47)**